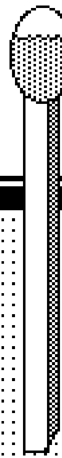
A GAME OF MATCHES



author

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Suppose that there are 30 matches on a table. I begin by picking up 1, 2, or 3 matches. Then my opponent must pick up 1, 2, or 3 matches. We continue in this fashion until the last match is picked up, and he who picks up that last match is the loser of the game.

What is the best strategy for playing this game?

That is, if there are x matches on the table, how many should I pick up?

Suppose that the loser pays $1.
Define the optimal value function

$f(x)$ = minimum cost if there are x matches remaining on the table, and it is your turn to remove matches.

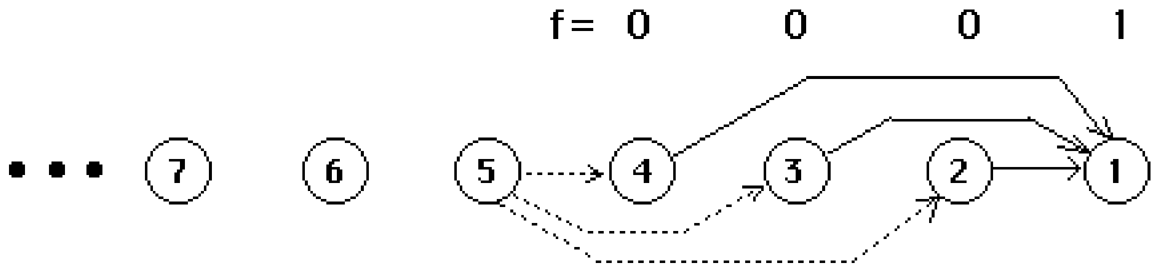
$d(x)$ = optimal number of matches to remove, if x matches remain on the table.

Recursive Definition of Optimal Value Function

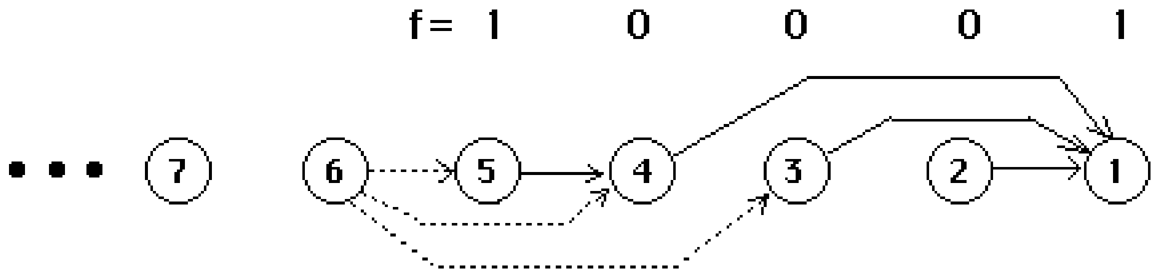
Assume that your opponent follows the strategy which is optimal for him. Then

$$\begin{cases} f(x) = \underset{\substack{d \in \{1,2,3\} \\ d \le x}}{\text{minimum}} \{ 1 - f(x-d) \}, & x = 2,3,4,5, \ldots 30 \\ f(1) = 1 \end{cases}$$

> If you remove **d** matches, **x−d** matches remain for your opponent; his optimal value is **f(x−d)**, and so your cost will be 1−f(x−d)



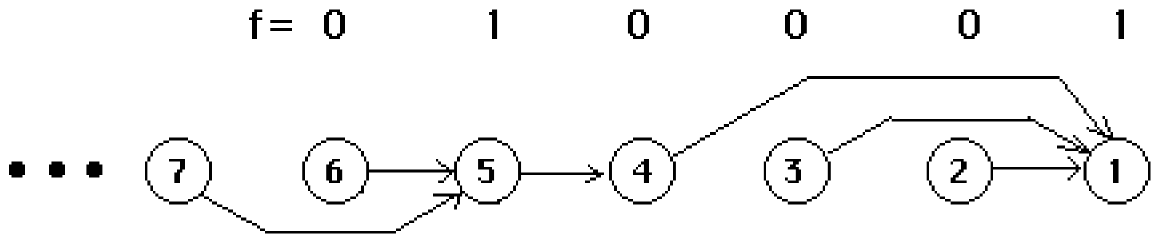
Suppose that 5 matches remain, and it is your turn....
no matter whether you remove 1, 2, or 3 matches, the number remaining for you opponent will be such that his optimal cost will be 0.



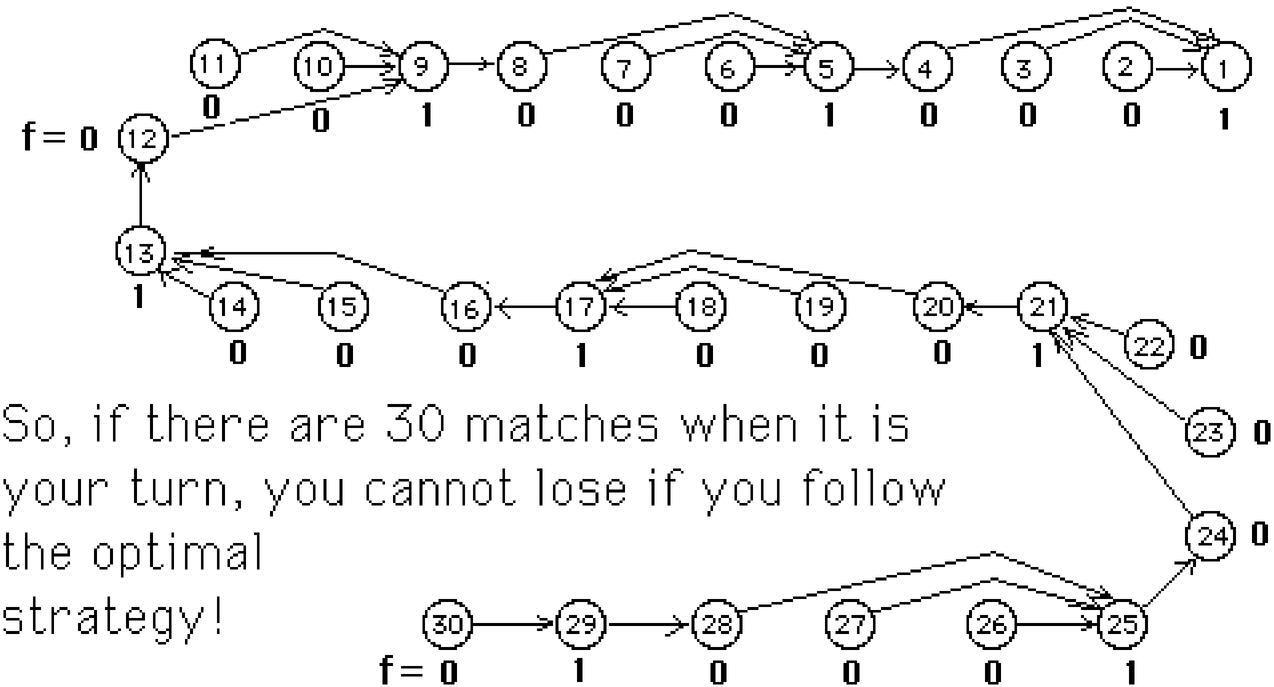
Suppose that 6 matches remain, and it is your turn...
Then your cost will be

f(6) = minimum {1−1, 1−0, 1−0} = minimum{0,1,1} = 0

and the optimal number of matches to remove is 1 (which leaves your opponent with 5 matches on the table)



Likewise, if there are 7 matches on the table when it is your turn, you should remove 2 matches so as to leave 5 on the table when it is your opponent's turn!



So, if there are 30 matches when it is your turn, you cannot lose if you follow the optimal strategy!