Dynamic Programming

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Characteristics of Dynamic Programming Applicatons

☞ The problem can be divided into ***stages***, with a ***decision*** required at each stage

☞ Each stage has a number of ***states*** associated with it. ("state" = information needed at that stage to make the optimal decision)

Characteristics of Dynamic Programming Applicatons

☞ The current state and the decision chosen at any stage determines either

the state at the next stage (deterministic DP)

or

the probability distribution of the state at the next stage (stochastic DP)

(transition function)

Characteristics of Dynamic Programming Applicatons

- Given the current state, the optimal decision for each of the remaining stages do not depend on the previously reached states or previously chosen decisions. (The *"Principle of Optimality"*)

- There is a ***recursive*** relationship which relates the total cost or reward earned in stages

  t, t+1, t+2, ... T

  to the total cost or reward earned in stages

  t+1, t+2, ... T

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- ☞ Probabilistic (Stochastic) Models

Deterministic Models

Probabilistic Models

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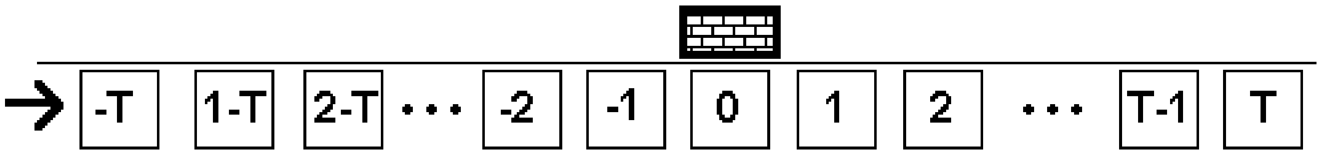
- ☞ The Casino Problem
- ☞ Optimal Batch Size Problem
- ☞ Machine Replacement with Random Failures

Example Applications of Dynamic Programming

Robert is trying to find a parking place near his favorite restaurant. He is approaching the restaurant from the west, and his goal is to park as nearby as possible. Robert is nearsighted and cannot see ahead... he can see only whether the space at his current location is empty.



When he arrives at an empty space, he must decide whether to park there or to continue to look for a closer space. Once he passes a space, he cannot return to it.

If Robert parks in space t, he incurs a cost $|t|$, i.e., the distance to the restaurant. If he dies not end up with a parking space, he is embarrassed and incurs a cost M (a big number)

He estimates that the probability that space t is empty is p_t

How can he use Dynamic Programming to develop a parking strategy that minimizes his expected cost?

Solve with $T=10$ *and* $p_t = \dfrac{|t|}{10}$

- At the beginning of each year, a firm observes its asset position d and may invest any amount x $(0 \le x \le d)$ in a risky investment.
- During each year, the money invested doubles with probability p and is completely lost with probability $1-p$.
- Independently of this investment, the firm's asset position increases by an amount y with probability q_y (y may be either positive or negative.)
- The firm initially has $10,000 in assets, and wants to maximize its expected asset position ten years from now.

What is the firm's optimal strategy to accomplish this?

AIR FLIGHT OVERBOOKING

- In the time interval between t and $t-1$ seconds before the departure of Braneast Airlines Flight 313, there is a probability p_t that the airline will receive a reservation for the flight (and probability $1-p_t$ that no reservations is received.)
- The flight can seat up to 100 passengers.
- At departure time, if r reservations have been accepted by the airline, there is a probability $q(y|r)$ that y passengers will show up for the flight.
- Each passenger who boards the flight adds $500 to the airline's revenues, but each passenger who shows up for the flight and cannot be seated receives $200 in compensation.

- Each passenger who boards the flight adds $500 to the airline's revenues, but each passenger who shows up for the flight and cannot be seated receives $200 in compensation.
- Thus, because of "no-shows", it is probably optimal for the airline to "overbook", i.e., to accept reservations for more than the number of available seats.
- Assume that no reservations are received more than 100,000 seconds before flight time.

Develop a DP model to enable to airline to maximize its expected revenue from Flight 313.

- At the beginning of each week, a machine is either running or broken down.
- If the machine runs throughout the week, it earns revenues of \$100, but if it breaks down anytime during the week, it earns NO revenues that week.
- If the machine is running at the beginning of the week, we may perform maintenance to lessen the chance of a breakdown

$$P\{\text{breakdown} \mid \text{no maintenance}\} = 70\%$$
$$P\{\text{breakdown} \mid \text{maintenance}\} = 40\%$$

- Maintenance costs \$20 each week.

- If the machine is broken down at the beginning of the week, then it must be either replaced or repaired.
- Repair costs $40, and there is a 40% chance that the repaired machine will break down during the week.
- Replacement costs $90, but the new machine is guaranteed to run throughout its first week of operation.

Develop a DP model to determine a policy for repair, replacement, & maintenance which will maximize the expected net profit earned over a 4-week time period.

A JOBSHOP SCHEDULING PROBLEM

- At 7 a.m., eight people leave their cars for repair at Harry's Auto Repair Shop.
- If person i's car is ready by time t (where t=0 is 7a.m.), Harry will be paid $r_i(t)$.

For example, if person 2's car must be ready by 2pm (i.e., t=7, then we may have $r_2(8) = 0$.

- Harry estimates that with probability $p_i(t)$ it will take t hours to repair car # i.

Develop a DP model that will enable Harry to maximize his expected revenue for the day (which ends at t=10 (5pm)).