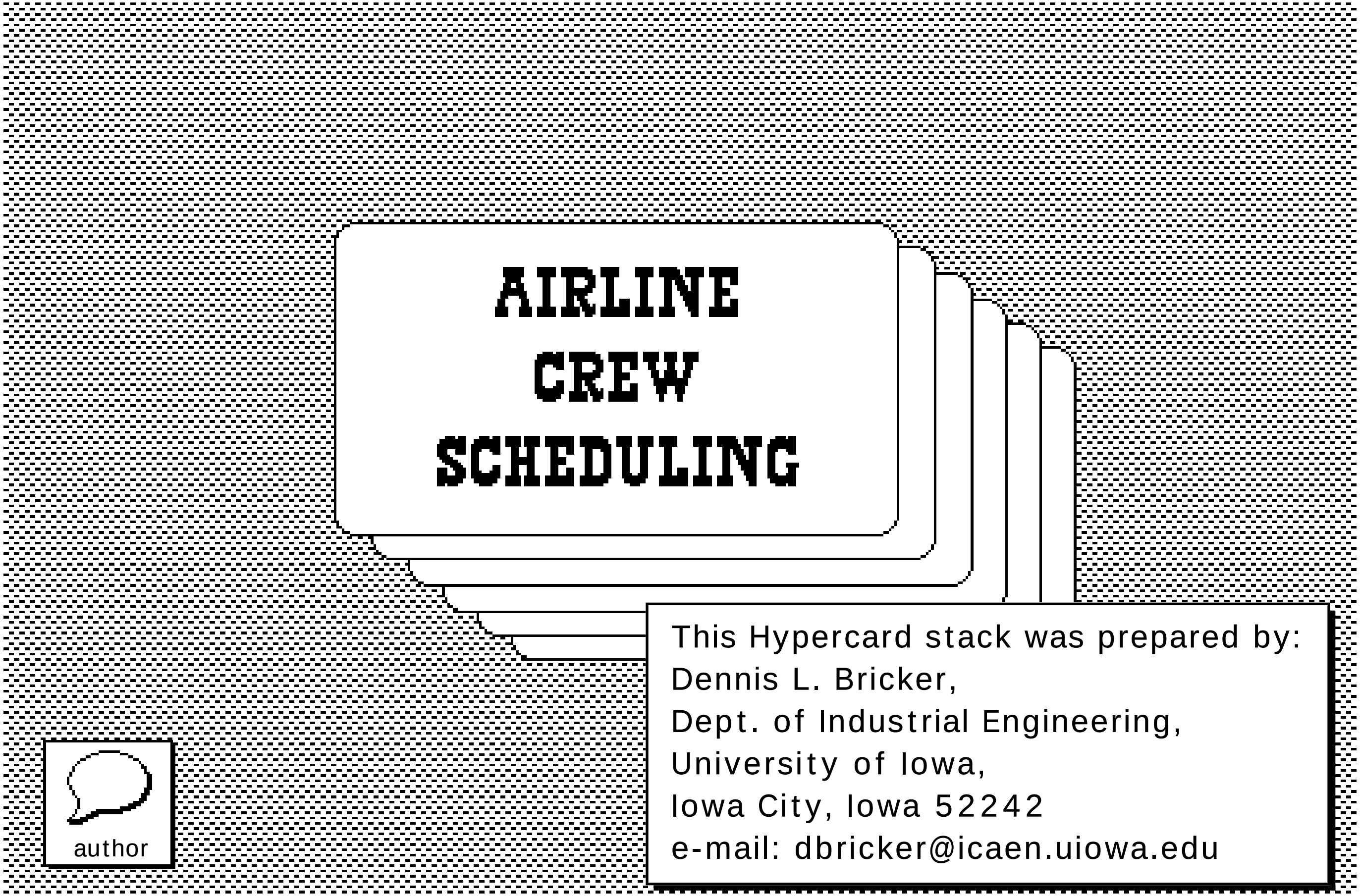
AIRLINE
CREW
SCHEDULING
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An airline operates 7 flights daily between New York & Chicago.

Crews are based in either city, and fly to-&-from the other city each day.

Flight #	Leave Chgo.	Arrive N.Y.	Flight #	Leave N.Y.	Arrive Chgo.
1	6 am	10 am	1	7 am	9 am
2	9 am	1 pm	2	8 am	10 am
3	noon	4 pm	3	10 am	noon
4	3 pm	7 pm	4	noon	2 pm
5	5 pm	9 pm	5	2 pm	4 pm
6	7 pm	11 pm	6	4 pm	6 pm
7	8 pm	midnight	7	6 pm	8 pm

- Each of airline's crews live either in Chicago or in New York
- Each day, a crew must fly one NY-Chgo and one Chgo-NY flight
- There must be at least 1 hour layover time between flights
- At end of the day, a crew must be in its home city.

- The airline wants to assign the flights to the crews to minimize total layover time
- How many crews should be based in each city?

$$X_{ij} = \begin{cases} 1 \text{ if a crew is assigned flight \#}i \text{ from Chgo-NY and flight \#}j \text{ from NY-Chgo.} \\ 0 \text{ otherwise} \end{cases}$$

For example, $X_{14} = 1$ indicates that a crew is assigned Chgo-NY Flight #1, departing at 6 AM & arriving at 10 AM. After a 2-hour layover, they return to Chicago on NY-Chgo Flight #4, departing at noon and arriving in Chicago at 2 PM.

Layover times for flight pairs

i \ j	j=1	2	3	4	5	6	7
i=1	∞	∞	∞	2	4	6	8
2	∞	∞	∞	∞	1	3	5
3	3	2	∞	∞	∞	∞	2
4	6	5	3	1	∞	∞	∞
5	8	7	5	3	1	∞	∞
6	10	9	7	5	3	1	∞
7	11	10	8	6	4	2	∞

these pairs flown by a Chicago-based crew

these pairs are not feasible

these pairs flown by a NY-based crew

Row Reduction

$$
\begin{matrix}
\infty & \infty & \infty & 2 & 4 & 6 & 8 \\
\infty & \infty & \infty & \infty & 1 & 3 & 5 \\
3 & 2 & \infty & \infty & \infty & \infty & 2 \\
6 & 5 & 3 & 1 & \infty & \infty & \infty \\
8 & 7 & 5 & 3 & 1 & \infty & \infty \\
10 & 9 & 7 & 5 & 3 & 1 & \infty \\
11 & 10 & 8 & 6 & 4 & 2 & \infty
\end{matrix}
\Rightarrow
\begin{matrix}
\infty & \infty & \infty & 0 & 2 & 4 & 6 \\
\infty & \infty & \infty & \infty & 0 & 2 & 4 \\
1 & 0 & \infty & \infty & \infty & \infty & 0 \\
5 & 4 & 2 & 0 & \infty & \infty & \infty \\
7 & 6 & 4 & 2 & 0 & \infty & \infty \\
9 & 8 & 6 & 4 & 2 & 0 & \infty \\
9 & 8 & 6 & 4 & 2 & 0 & \infty
\end{matrix}
$$

Column Reduction

$$
\begin{matrix}
\infty & \infty & \infty & 0 & 2 & 4 & 6 \\
\infty & \infty & \infty & \infty & 0 & 2 & 4 \\
1 & 0 & \infty & \infty & \infty & \infty & 0 \\
5 & 4 & 2 & 0 & \infty & \infty & \infty \\
7 & 6 & 4 & 2 & 0 & \infty & \infty \\
9 & 8 & 6 & 4 & 2 & 0 & \infty \\
9 & 8 & 6 & 4 & 2 & 0 & \infty
\end{matrix}
\quad \Rightarrow \quad
\begin{matrix}
\infty & \infty & \infty & 0 & 2 & 4 & 6 \\
\infty & \infty & \infty & \infty & 0 & 2 & 4 \\
0 & 0 & \infty & \infty & \infty & \infty & 0 \\
4 & 4 & 0 & 0 & \infty & \infty & \infty \\
6 & 6 & 2 & 2 & 0 & \infty & \infty \\
8 & 8 & 4 & 4 & 2 & 0 & \infty \\
8 & 8 & 4 & 4 & 2 & 0 & \infty
\end{matrix}
$$

∞	∞	∞	0	2	4	6
∞	∞	∞	∞	0	2	4
0	0	∞	∞	∞	∞	0
4	4	0	0	∞	∞	∞
6	6	2	2	0	∞	∞
8	8	4	4	2	0	∞
8	8	4	4	2	0	∞

The zeroes can be covered by only five lines!

Therefore, the matrix reductions are not complete.

"Mixed Reduction"

$$
\begin{matrix}
\infty & \infty & \infty & 0 & 2 & 4 & 6 \\
\infty & \infty & \infty & \infty & 0 & 2 & 4 \\
0 & 0 & \infty & \infty & \infty & \infty & 0 \\
4 & 4 & 0 & 0 & \infty & \infty & \infty \\
6 & 6 & 2 & 2 & 0 & \infty & \infty \\
8 & 8 & 4 & 4 & 2 & 0 & \infty \\
8 & 8 & 4 & 4 & 2 & 0 & \infty
\end{matrix}
\quad \Longrightarrow \quad
\begin{matrix}
\infty & \infty & \infty & 0 & 2 & 4 & 2 \\
\infty & \infty & \infty & \infty & 0 & 2 & 0 \\
0 & 0 & \infty & \infty & \infty & \infty & 0 \\
0 & 0 & 0 & 0 & \infty & \infty & \infty \\
2 & 2 & 2 & 2 & 0 & \infty & \infty \\
4 & 4 & 4 & 4 & 2 & 0 & \infty \\
4 & 4 & 4 & 4 & 2 & 0 & \infty
\end{matrix}
$$

$$
\begin{matrix}
\infty & \infty & \infty & 0 & 2 & 4 & 2 \\
\infty & \infty & \infty & \infty & 0 & 2 & 0 \\
0 & 0 & \infty & \infty & \infty & \infty & 0 \\
0 & 0 & 0 & 0 & \infty & \infty & \infty \\
2 & 2 & 2 & 2 & 0 & \infty & \infty \\
4 & 4 & 4 & 4 & 2 & 0 & \infty \\
4 & 4 & 4 & 4 & 2 & 0 & \infty
\end{matrix}
$$

Only six lines are required to cover all the zeroes!

Therefore, the matrix reductions are not complete.

"Mixed Reduction"

$$
\begin{matrix}
\infty & \infty & \infty & 0 & 2 & 4 & 2 \\
\infty & \infty & \infty & \infty & 0 & 2 & 0 \\
0 & 0 & \infty & \infty & \infty & \infty & 0 \\
0 & 0 & 0 & 0 & \infty & \infty & \infty \\
2 & 2 & 2 & 2 & 0 & \infty & \infty \\
4 & 4 & 4 & 4 & 2 & 0 & \infty \\
4 & 4 & 4 & 4 & 2 & 0 & \infty
\end{matrix}
\quad \Longrightarrow \quad
\begin{matrix}
\infty & \infty & \infty & 0 & 2 & 4 & 2 \\
\infty & \infty & \infty & \infty & 0 & 2 & 0 \\
0 & 0 & \infty & \infty & \infty & \infty & 2 \\
0 & 0 & 0 & 2 & \infty & \infty & \infty \\
0 & 0 & 0 & 2 & 0 & \infty & \infty \\
2 & 2 & 2 & 4 & 2 & 0 & \infty \\
2 & 2 & 2 & 4 & 2 & 0 & \infty
\end{matrix}
$$

∞	∞	∞	0	2	4	2
∞	∞	∞	∞	0	2	0
0	0	∞	∞	∞	∞	2
0	0	0	2	∞	∞	∞
0	0	0	2	0	∞	∞
2	2	2	4	2	0	∞
2	2	2	4	2	0	∞

Only six lines are required to cover all the zeroes!

Therefore, the matrix reductions are not complete.

"Mixed Reduction"

$$
\begin{matrix}
\infty & \infty & \infty & 0 & 2 & 4 & 2 \\
\infty & \infty & \infty & \infty & 0 & 2 & 0 \\
0 & 0 & \infty & \infty & \infty & \infty & 2 \\
0 & 0 & 0 & 2 & \infty & \infty & \infty \\
0 & 0 & 0 & 2 & 0 & \infty & \infty \\
2 & 2 & 2 & 4 & 2 & 0 & \infty \\
2 & 2 & 2 & 4 & 2 & 0 & \infty
\end{matrix}
\quad \Rightarrow \quad
\begin{matrix}
\infty & \infty & \infty & 0 & 2 & 6 & 2 \\
\infty & \infty & \infty & \infty & 0 & 4 & 0 \\
0 & 0 & \infty & \infty & \infty & \infty & 2 \\
0 & 0 & 0 & 2 & \infty & \infty & \infty \\
0 & 0 & 0 & 2 & 0 & \infty & \infty \\
0 & 0 & 0 & 2 & 0 & 0 & \infty \\
0 & 0 & 0 & 2 & 0 & 0 & \infty
\end{matrix}
$$

∞	∞	∞	0	2	6	2
∞	∞	∞	∞	0	4	0
0	0	∞	∞	∞	∞	2
0	0	0	2	∞	∞	∞
0	0	0	2	0	∞	∞
0	0	0	2	0	0	∞
0	0	0	2	0	0	∞

The zeroes cannot be covered by fewer than seven lines!

Therefore, we should be able to find a zero-cost assignment.

$$\begin{array}{ccccccc}
\infty & \infty & \infty & \boxed{0} & 2 & 6 & 2 \\
\infty & \infty & \infty & \infty & 0 & 4 & 0 \\
0 & 0 & \infty & \infty & \infty & \infty & 2 \\
0 & 0 & 0 & 2 & \infty & \infty & \infty \\
0 & 0 & 0 & 2 & 0 & \infty & \infty \\
0 & 0 & 0 & 2 & 0 & 0 & \infty \\
0 & 0 & 0 & 2 & 0 & 0 & \infty
\end{array}$$

We look for a row or a column with a single zero...

... this indicates a necessary assignment

$$
\begin{matrix}
\infty & \infty & \infty & \boxed{0} & 2 & 0 & 2 \\
\infty & \infty & \infty & \infty & 0 & 4 & \boxed{0} \\
0 & 0 & \infty & \infty & \infty & \infty & 2 \\
0 & 0 & 0 & 2 & \infty & \infty & \infty \\
0 & 0 & 0 & 2 & 0 & \infty & \infty \\
0 & 0 & 0 & 2 & 0 & 0 & \infty \\
0 & 0 & 0 & 2 & 0 & 0 & \infty
\end{matrix}
$$

∞	∞	∞	$\boxed{0}$	2	0	2
∞	∞	∞	∞	0	4	$\boxed{0}$
$\boxed{0}$	0	∞	∞	∞	∞	2
0	$\boxed{0}$	0	2	∞	∞	∞
0	0	$\boxed{0}$	2	0	∞	∞
0	0	0	2	$\boxed{0}$	0	∞
0	0	0	2	0	$\boxed{0}$	∞

Total cost = 25 hours
(= total layover time)

(there are several optimal solutions)

Optimal Solution

2 Chicago-based crews, flying

Flight #1 to NY, then Flight #4 to Chgo.

Flight #2 to NY, then Flight #7 to Chgo.

5 New-York-based crews, flying

Flight #1 to Chgo, then Flight #3 to NY

Flight #2 to Chgo, then Flight #4 to NY

Flight #3 to Chgo, then Flight #5 to NY

Flight #5 to Chgo, then Flight #6 to NY

Flight #6 to Chgo, then Flight #7 to NY