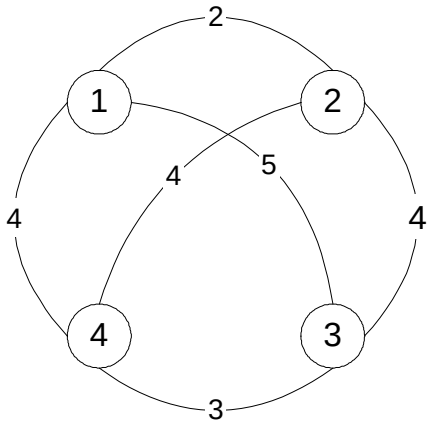
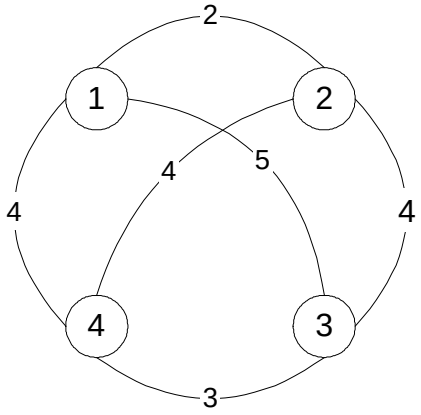
Name:

56:272 Integer Programming & Network Flows
Quiz #8 – Fall 2003

A traveling salesman has his home office at city #1 and must visit each of cities #2, 3, & 4 exactly once, and then return to his home office.





He formulates the problem of finding the shortest tour by the integer programming model:

Minimize $\sum_i \sum_j D_{ij} X_{ij}$

s.t. $\sum_i \sum_j X_{ij} = 1$ *forall* $i = 1,\ldots 4$ *(i.e., a city must follow each city #i)*

$\sum_i \sum_j X_{ij} = 1$ *forall* $j = 1,\ldots 4$ *(i.e., a city must precede each city #j)*

$X_{ij} \in \{0,1\}$ *forall* i & j

a. By inspection, indicate the solution on the diagram on the ***left*** above.

b. Is the solution a ***minimum spanning one-tree***? ____ If not, find the minimum spanning one-tree (with city #1 as the root, i.e., it is added last to the tree with two connecting edges). Indicate this on the map on the ***right***.

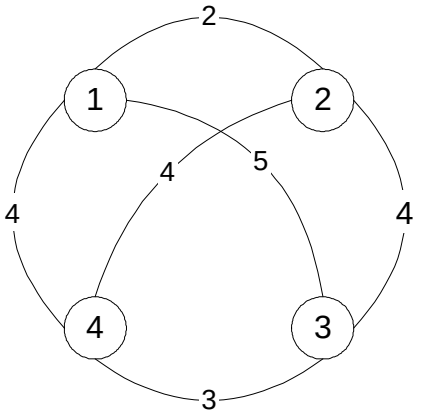
c. ***Vertex Penalty Method:*** Using penalties of magnitude 1, which penalties should be assigned to each of the four vertices (cities)?

City	1	2	3	4
Penalty				

d. What are the distances now used to re-compute the minimum spanning one-tree?

	1	2	3	4
1				
2				
3				
4				

e. Indicate the new minimum spanning one-tree on the diagram on the right.



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f. ***Multi-commodity flow model of TSP.*** For each of the three cities to be visited, define a commodity $Y^k (k = 1, 2, 3)$ to be delivered to that city. Complete the constraints below for commodity #2 which would be added to the TSP formulation (together with the usual assignment constraints). (Complete the blanks and circle the ≤, =, or ≥.)

$Y_{12}^2 + Y_{13}^2 + Y_{14}^2 \; (\leq, =, \geq) \_\_\_\_$ $\qquad$ $Y_{12}^2 + Y_{32}^2 + Y_{42}^2 = \_\_\_\_$

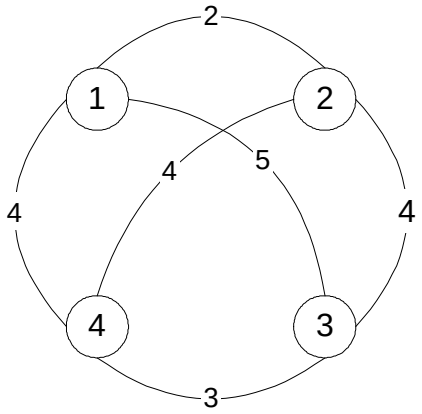
$Y_{31}^2 + Y_{32}^2 + Y_{34}^2 \; (\leq, =, \geq) Y_{13}^2 + Y_{23}^2 + Y_{43}^2$ $\qquad$ $Y_{41}^2 + Y_{42}^2 + Y_{43}^2 \; (\leq, =, \geq) Y_{14}^2 + Y_{24}^2 + Y_{34}^2$

$$Y_{12}^2 (\leq, =, \geq) X_{12}, \;\; Y_{13}^2 (\leq, =, \geq) X_{13}, \;\; \textit{etc.}$$

g. ***Nearest Insertion Heuristic.*** What is the order in which the cities would be added to the tour, beginning with node #1, if the <u>nearest</u> insertion heuristic is used?

Order	#1	#2	#3	#4
City #	1			

Show the tour found on the diagram on the right.



h. ***Farthest Insertion Heuristic.*** What is the order in which the cities would be added to the tour, beginning with node #1, if the <u>farthest</u> insertion heuristic is used?

Order	#1	#2	#3	#4
City #	1			

Show the tour found on the diagram on the right.