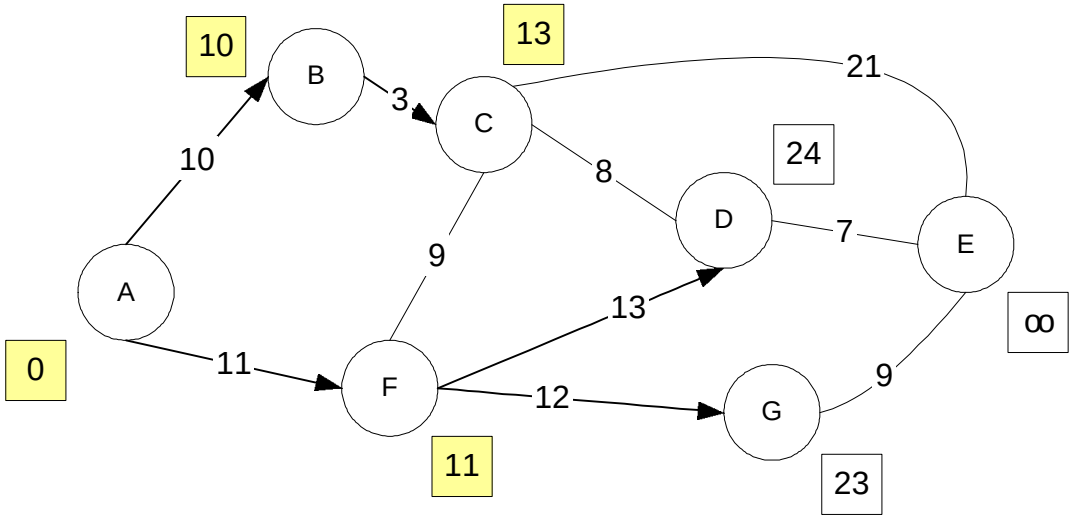
56:272 Integer Programming & Network Flows
Quiz #5--Fall 2003

Part One: Shortest Paths

a. The first step of the shortest-path algorithm presented in class is to
- ❑ select the node which is to be the origin of the path
- ❑ select the node which is to be the destination of the path
- ❑ select any arbitrary node
- ❑ none of the above

b. The "label" which is assigned to a node represents
- ❑ the order in which that node appears on the shortest path
- ❑ the cost of adding that node to the path
- ❑ the length of the shortest path from the origin to that node
- ❑ none of the above

Indicate **+** for *true*, **O** for *false*:
___c. The "label" which is assigned to the origin of the path is initially temporary.
___d. At the end of the shortest-path algorithm, the length of the shortest path to *every* node has been computed.



Consider the network above. The shortest-path algorithm is being applied in order to find the shortest path from node A to node E. The boxes are labels, with the shaded boxes represent those which are permanent.

e. The node whose label was most recently made permanent is
circle: A B C F

f. After the next iteration, the labels of the nodes will be… *(you need only show those which changed)*
A: ___ B: ___ C: ___ D: ___ E: ___ F: ___ G: ___

g. After the next iteration, the nodes with permanent labels will be:
circle: A B C D E F G

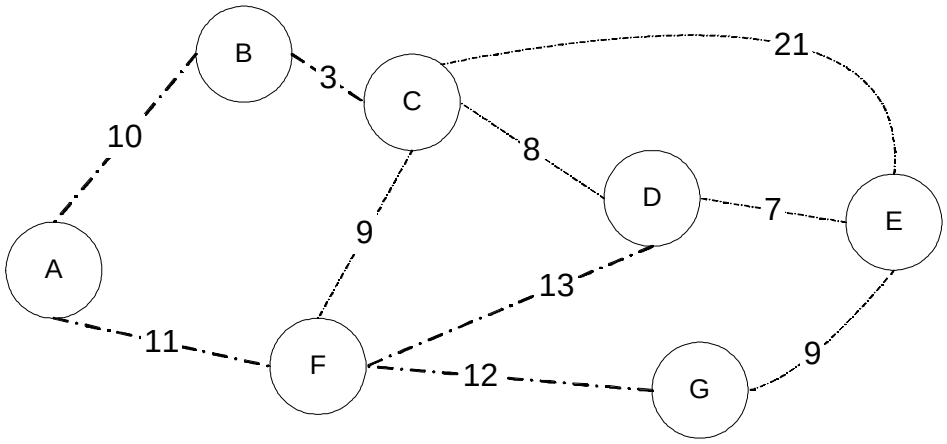
Note: Part Two is on reverse side!

Part Two: Minimum Spanning Trees (MST)

a. The first step of the MST algorithm presented in class is to
- ❑ select the node with the greatest distance between it and any other node
- ❑ select the node with the smallest distance between it and any other node
- ❑ select the node that is nearest the origin
- ❑ select any arbitrary node
- ❑ none of the above

b. The following *(one or more)* statement(s) is/are true of a tree
- ❑ the number of nodes and links (edges) are equal
- ❑ the number of links is one less than the number of nodes
- ❑ the number of links is one greater than the number of nodes
- ❑ between two nodes there is exactly one path
- ❑ there exists a cycle of nodes

Indicate **+** for *true*, **O** for *false*:
__c. Removing an edge from a tree will result in two trees.
__d. All nodes of a network must be included in its spanning tree.
__e. Every spanning tree of a given network contains the same number of nodes.
__f. Every spanning tree of a given network contains the same number of edges.



A new residential area is being developed, and the nodes A through G above represent homes. The edges represent the cost of joining the homes by a new water main (assuming the cost does not depend upon the amount of water which must pass through the pipe.)

g. If node E is already joined to the water system (pipe not shown), indicate which other pipes should be put into place so that all homes are connected to the water system at minimum cost.

h. Is there more than one optimal solution to this problem? |_| yes |_| no

i. If, rather than node E, node A were now connected to the main water source, the optimal cost will be
- ❑ greater
- ❑ smaller
- ❑ unchanged
- ❑ none of the above