56:272 Integer Programming & Network Flows
Final Exam Solutions -- Fall '99

• *Write your name on the first page, and initial the other pages.*
• *Answer all the multiple-choice questions and X of the remaining questions.*

		Possible
Part A: Do ALL!	Multiple choice	35
Part B: Select 4		
	1. Traveling salesman	15
	2. Knapsack problem via DP	15
	3. Knapsack problem via branch-&-bound	15
	4. Benders decomposition	15
	5. Lagrangian relaxation/duality	15
Part C: Integer LP Model Formulation -- Select 2		
	1. Traffic monitoring device location	10
	2. Mailbox location	10
	3. Cassette tape allocation	10
Part D: Integer LP Model Formulation -- Select 2		
	1. Black Box Company	10
	2. Dandy Diesel Company	10
	3. Top T-shirt Company	10
Total:		145

❍ ❍ ❍ ❍ ❍ ❍ **PART A** ❍ ❍ ❍ ❍ ❍ ❍ ❍

Multiple Choice *If more than one answer is possible, either is acceptable.*

_a_ 1. When using the Hungarian method to solve assignment problems, if the number of lines drawn to cover the zeroes in the reduced matrix is larger than the number of rows, ...
- $\boxed{\text{a.}}$ a mistake has been made, and one should review previous steps.
- b. this indicates no solution exists.
- c. this means an optimal solution has been reached.
- d. *none of the above*.

_d__ 2. A matrix which is totally unimodular ...
- a. must be square.
- b. must be lower triangular.
- c. has a determinant equal to zero.
- $\boxed{\text{d.}}$ *none of the above*.

_c__ 3. A node-arc incidence matrix of a graph with n nodes ...
- a. has n−1 rows.
- b. is a square matrix.
- $\boxed{\text{c.}}$ has rank n−1.
- d. *none of the above*.

a&b 4. The adjacency matrix of an undirected graph with n nodes ...
- $\boxed{\text{a.}}$ is a square matrix.
- $\boxed{\text{b.}}$ is a symmetric matrix.
- c. has a determinant equal to ±1.
- d. *none of the above*.

_c_ 5. Balas' algorithm is referred to as the "Additive Algorithm" because ...
- a. the objective is the sum of (nonnegative) costs.
- b. variables are added one at a time to the set of fixed variables.
- $\boxed{\text{c.}}$ no multiplications or divisions are required
- d. *none of the above*.

_b_ 6. Floyd's algorithm for a graph with n nodes ...
- a. finds the shortest paths from a source node to each of the other nodes.
- $\boxed{\text{b.}}$ requires exactly n iterations to be performed.

c. is a specialized simplex algorithm.
d. *none of the above*.

_b_ 7. The vertex penalty method for the traveling salesman problem ...
a. adds penalties to the vertices within a subtour found by the assignment problem.
b. is an example of Lagrangian relaxation.
c. may be used to compute an upper bound.
d. *none of the above*.

_d_ 8. In simulated annealing, ...
a. the probability of accepting a step which results in an improvement increases at each iteration.
b. nodes are added to TSP subtours until all nodes are included.
c. the objective function improves at each iteration.
d. *none of the above*.

_a_ 9. Subtour elimination constraints for the traveling salesman problem ...
a. are appended to the assignment problem constraints to model the TSP
b. are appended to conservation of flow constraints to model the TSP
c. are applicable only to symmetric problems.
d. *none of the above*.

_a_ 10. If the current solution of the transportation problem is degenerate...
a. the reduced cost of at least one zero shipment is zero.
b. the number of sources must be equal to the number of destinations.
c. the next iteration will produce no improvement in the objective function
d. *none of the above*.

_a_ 11. An optimal solution of a traveling salesman problem (in a complete undirected network) is always...
a. a Hamiltonian tour
b. a Lagrangian tour
c. an Euler tour
d. *none of the above*.

_c_ 12. An optimal solution of the Chinese postman problem (in an undirected network) is always
a. a Hamiltonian tour
b. a Lagrangian tour
c. an Euler tour
d. *none of the above*.

_a_ 13. A simple plant location problem...
a. places no limits on the plant capacities.
b. is also referred to as the p-median problem.
c. places a limit on the values of the plant capacities (if built).
d. *none of the above*.

_b_ 14. When applying Benders' method to the capacitated plant location problem, the "master" problem...
a. evaluates the total cost if a specified set of plants are open
b. selects the next trial set of plants to be open
c. gives an upper bound on the cost of the optimal solution
d. *none of the above*.

_b_ 15. The quadratic assignment problem...
a. includes quadratic constraints.
b. has the same constraints as the original assignment problem.
c. includes X_{ij}^2 terms in the objective function.
d. is a specialized form of the "generalized assignment problem" (GAP).
e. *none of the above*.

_d_ 16. The generalized assignment problem...
a. includes the original assignment constraints, plus some additional constraints.
b. can be solved by the Hungarian algorithm together with branch-and-bound
c. includes the transportation problem as a special case.
d. *none of the above*.

_c_ 17. A genetic algorithm for the line-balancing problem with N tasks to be assigned to stations ...

a. if it converges, guarantees that the solution is optimal.
b. uses a population size equal to N.
$\boxed{\text{c.}}$ represents an individual within the population by a string of numbers of length N.
d. *none of the above*.

_a_ 18. Weber's problem…

$\boxed{\text{a.}}$ has a nonlinear objective function
b. is to find the median of a network
c. is an integer LP
d. *none of the above*

_a_ 19. The adjacency matrix of an directed graph with n nodes ...

$\boxed{\text{a.}}$ is a square matrix.
b. is a symmetric matrix.
c. has +1, −1, and 0 as entries.
d. *none of the above*.

_c__ 20. The LP formulation of the problem to find the shortest path in a network ...

a. has right-hand-sides which are all zero.
b. may require branch-and-bound if the LP solution is not integer.
$\boxed{\text{c.}}$ has a dual LP which finds the longest path in a network.
d. *none of the above*.

_c_ 21. The LP model for an $n \times n$ linear assignment problem...

a. has an integer optimal solution only if the costs are integer.
b. has 2n basic variables.
$\boxed{\text{c.}}$ has only degenerate basic feasible solutions.
d. *none of the above*

_c_ 22. The following is true of an n-item zero-one knapsack problem with integer values for the weights, item values, and capacity...

a. when solving by branch-&-bound, the # of terminal nodes in the complete enumeration tree is n^2
b. in the DP model, the state variable has 2^n possible values
$\boxed{\text{c.}}$ in the DP model, the number of stages is n
d. *none of the above*

_b__ 23. Johnson's algorithm is to solve…

a. assembly-line balancing problems
$\boxed{\text{b.}}$ flowshop scheduling problems
c. traveling salesman problems
d. *none of the above*

_d_ 24. The "integrality property" of a Lagrangian relaxation…

a. implies that the bound on the primal solution is superior to that of the LP relaxation
b. implies that the optimal values of the Lagrangian multipliers are integer
c. implies that the duality gap is zero, i.e., the primal optimum = dual optimum
$\boxed{\text{d.}}$ *none of the above*

_a_ 25. Gomory's cutting plane method discussed in this class…

$\boxed{\text{a.}}$ stops when it finds a feasible integer solution, guaranteeing optimality
b. is not limited to problems in which all variables are required to be integer
c. requires that the problem be stated in a standard form (minimization with $\leq$ constraints)
d. *none of the above*

_c_ 26. The subproblem of Benders' decomposition algorithm applied to the capacitated plant location problem…

a. finds solutions which, if feasible, must be optimal.
b. produces a lower bound on the optimal value of the original problem.
$\boxed{\text{c.}}$ produces an upper bound on the optimal value of the original problem.
d. *none of the above*

_c_ 27. A minimum spanning tree of an undirected network with n nodes ...

a. can be given a strongly-connected orientation.
b. contains no nodes of degree 2
$\boxed{\text{c.}}$ has n−1 edges.
d. *none of the above*

_a_ 28. Vogel's Approximation Method (VAM)...
$\boxed{\text{a.}}$ always yields a basic feasible solution of a transportation problem.
b. cannot be applied to an assignment problem, because of degeneracy.
c. will never result in a degenerate solution.
d. *none of the above*

a&b 29. Djikstra's algorithm for a graph with n nodes ...
$\boxed{\text{a.}}$ finds the shortest paths from a source node to each of the other nodes.
$\boxed{\text{b.}}$ requires exactly n iterations to be performed.
c. is a special case of the simplex algorithm for LP.
d. *none of the above*

_c_ 30. Floyd's algorithm is used to find…
a. the maximum flow in a network
b. a minimum spanning tree
$\boxed{\text{c.}}$ shortest paths in a network
d. *none of the above*

_c_ 31. The Chinese postman problem in an *undirected* network…
a. can be solved by solving a transportation problem
b. requires that all nodes have zero polarity
$\boxed{\text{c.}}$ has a solution ≥ than that of the corresponding traveling salesman problem
d. *none of the above*

32. Check the problems in the list below which are known to have polynomial-time complexity, (i.e., are in the set P):

___Traveling salesman problem	_X_ Shortest path problem
_X_ Minimum spanning tree problem	___Generalized assignment problem
___Simple plant location problem	___ Capacitated plant location problem
___Knapsack problem	___ Set-covering problem
_X_ Transportation problem	_X_ Linear assignment problem

❍ ❍ ❍ ❍ ❍ ❍ ❍ **PART B** ❍ ❍ ❍ ❍ ❍ ❍

B1. Traveling Salesman Problem. Five products are to be manufactured weekly on the same machine. The table below gives the cost of switching the machine from one product to another product. (Assume that this is also the cost of switching to the last product of the week to the first product to be scheduled the following week!)

to: \ from:	A	B	C	D	E
A	∞	3	6	7	5
B	3	∞	2	8	1
C	5	3	∞	6	2
D	1	3	5	∞	1
E	4	1	3	8	∞

a. The nearest neighbor heuristic, starting with product A, yields the product sequence _A-B-E-C-D-A_ with cost _14_.

After applying the "Hungarian Method" to the above matrix to solve the associated assignment problem (with large number, ∞, inserted along the diagonal), we have:

to: \ from:	A	B	C	D	E
A	∞	0	2	$\boxed{0}$	3

B	2	∞	$\boxed{0}$	3	0
C	3	1	∞	0	$\boxed{0}$
D	$\boxed{0}$	2	3	∞	0
E	3	$\boxed{0}$	1	3	∞

b. What is the solution of this assignment problem? A=>D=>A, B=>C=>E=>B
c. What is its cost? ___13_____
d. Is it a valid product sequence? __NO___ If not, why not? it consists of two subtours
e. If not a valid sequence, what bound on the optimal cost does this result provide?
(circle: upper / $\boxed{\text{lower}}$)
f. If *not* a valid sequence, what single constraint might be added to the assignment problem to eliminate the solution which you have obtained (but not eliminate any valid sequence)?

Several constraints might be used, e.g.,
the sum of edges in the subgraph corresponding to a subtour of length k must be no more than k-1:

$X_{DA}+X_{AD}\leq 1,\ X_{BC} + X_{CE} + X_{EB} \leq 2$

the sum of edges from one of two disconnected subgraphs must be at least 1:

$X_{AB} + X_{AC} + X_{AE} + X_{DB} + X_{DC} + X_{DE} \geq 1$

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B2. DP Solution of Knapsack Problem: A "knapsack" is to be filled to maximize the value of the contents, subject to a weight restriction of 12 kg.:

Item #	Weight (kg)	Value ($)
1	5	9
2	3	4
3	6	10
4	4	6
5	2	3
6	1	2

At most one unit of any item is to be included. A *dynamic programming* model was defined: we imagine that we consider the items one at a time, starting with item (stage) 6 and ending with item (stage) 1. We define the state s_i to be the capacity remaining in the knapsack when item *i* is considered, and $x \in \{0,1\}$ to be the state variable. The optimal value function is defined as:

$f_i(s)$ = maximum value of a knapsack consisting of items i, i-1, …2, 1 if s units of capacity remain to be filled, i=6, 5, 4, 3, 2, 1

where $f_0(s) = 0$ for any $s \geq 0$

and we wish to determine $f_6(12)$.

The computation was done recursively by considering at each stage every combination of the state and decision variables (where $-\infty$ represents an infeasible combination of s & x):

```
---Stage 1---
        s \ x:   0        1

        0 |   0.0000       -∞
        1 |   0.0000       -∞
        2 |   0.0000       -∞
        3 |   0.0000       -∞
        4 |   0.0000       -∞
        5 |   0.0000   9.0000
        6 |   0.0000   9.0000
        7 |   0.0000   9.0000
        8 |   0.0000   9.0000
```

9	0.0000	9.0000
10	0.0000	9.0000
11	0.0000	9.0000
12	0.0000	9.0000

.
.
. etc.
.

---Stage 5---

s \ x:	0	1
0	0.0000	$-\infty$
1	0.0000	$-\infty$
2	0.0000	3.0000
3	4.0000	3.0000
4	6.0000	3.0000
5	9.0000	7.0000
6	10.0000	9.0000
7	10.0000	12.0000
8	13.0000	13.0000
9	15.0000	13.0000
10	16.0000	16.0000
11	19.0000	18.0000
12	19.0000	19.0000

---Stage 6---

s \ x:	0	1	
0	0.0000	$-\infty$	
1	0.0000	2.0000	
2	3.0000	2.0000	
3	4.0000	5.0000	
4	6.0000	6.0000	
5	9.0000	8.0000	
6	10.0000	11.0000	
7	12.0000	12.0000	
8	$\boxed{???????}$	14.0000	← missing value is 13 (= 0 + $f_5(8-0)$)
9	15.0000	15.0000	
10	16.0000	17.0000	
11	19.0000	18.0000	
12	19.0000	21.0000	

At each stage, for every possible value of s the optimal value and decision were determined:

Stage 6:

State s	Optimal Values $f_6(s)$	Optimal Decisions	Resulting State	
0	0.0000	0	0	
1	2.0000	1	0	
2	3.0000	0	2	
3	5.0000	1	2	
4	6.0000	0	4	
		1	3	
5	$\boxed{9.0}$	$\boxed{0}$	$\boxed{5}$	← missing values!
6	11.0000	1	5	
7	12.0000	0	7	
		1	6	
8	14.0000	1	7	
9	15.0000	0	9	
		1	8	
10	17.0000	1	9	
11	19.0000	0	11	

12	21.0000	1	11

Stage 5:

State s	Optimal Values $f_5(s)$	Optimal Decisions	Resulting State
0	0.0000	0	0
1	0.0000	0	1
2	3.0000	1	0
3	4.0000	0	3
4	6.0000	0	4
5	9.0000	0	5
6	10.0000	0	6
7	12.0000	1	5
8	13.0000	0	8
		1	6
9	15.0000	0	9
10	16.0000	0	10
		1	8
11	19.0000	0	11
12	19.0000	0	12
		1	10

Stage 4:

State s	Optimal Values $f_4(s)$	Optimal Decisions	Resulting State
0	0.0000	0	0
1	0.0000	0	1
2	0.0000	0	2
3	4.0000	0	3
4	6.0000	1	0
5	9.0000	0	5
6	10.0000	0	6
7	10.0000	0	7
		1	3
8	13.0000	0	8
9	15.0000	1	5
10	16.0000	1	6
11	19.0000	0	11
12	19.0000	0	12
		1	8

Stage 3:

State s	Optimal Values $f_3(s)$	Optimal Decisions	Resulting State
0	0.0000	0	0
1	0.0000	0	1
2	0.0000	0	2
3	4.0000	0	3
4	4.0000	0	4
5	9.0000	0	5
6	10.0000	1	0
7	10.0000	1	1
8	13.0000	0	8
9	14.0000	1	3
10	14.0000	1	4
11	19.0000	1	5
12	19.0000	1	6

Stage 2:

```
          Optimal        Optimal    Resulting
 State s  Values f2(s)   Decisions     State
        0   0.0000         0             0
        1   0.0000         0             1
        2   0.0000         0             2
        3   4.0000         1             0
        4   4.0000         1             1
        5   9.0000         0             5
        6   9.0000         0             6
        7   9.0000         0             7
        8  13.0000         1             5
        9  13.0000         1             6
       10  13.0000         1             7
       11  13.0000         1             8
       12  13.0000         1             9

------------------------------------------------------------
Stage 1:
          Optimal        Optimal    Resulting
 State s  Values f1(s)   Decisions     State
        0   0.0000         0             0
        1   0.0000         0             1
        2   0.0000         0             2
        3   0.0000         0             3
        4   0.0000         0             4
        5   9.0000         1             0
        6   9.0000         1             1
        7   9.0000         1             2
        8   9.0000         1             3
        9   9.0000         1             4
       10   9.0000         1             5
       11   9.0000         1             6
       12   9.0000         1             7
```

a. Formulate this problem as a 0-1 ILP problem.

$$\boxed{\begin{array}{l} \text{Max } 9X1 + 4X2 + 10X3 + 6X4 + 3X5 + 2X6 \\ \text{s.t. } 5X1 + 3X2 + 6X3 + 4X4 + 2X5 + X6 \leq 12 \\ \quad Xj \in \{0, 1\} \text{ for } i=1,2,\ldots 6 \end{array}}$$

b. There are four values missing in the tables above. Indicate their values. *(See tables)*

c. What is the optimal value for this problem, i.e., $f_6(12)$? _21_

d. What are the optimal contents of the knapsack? items 1, 3, & 6

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B3. Branch-&-Bound Solution of Knapsack Problem. Consider the 0-1 knapsack problem in the previous problem, except that the maximum capacity is limited to 11 kg.:

Item #	Weight (kg)	Value ($)	ratio ($/kg)
1	5	9	1.8
2	3	4	1.333
3	6	10	1.667
4	4	6	1.6
5	2	3	1.5
6	1	2	2

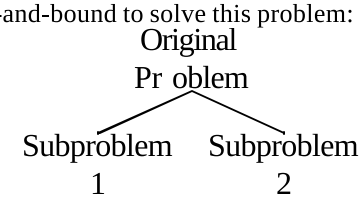
a. What is the solution of the LP relaxation of this problem?

Item #i	Xi
1	1
2	
3	5/6
4	
5	
6	1

b. Objective function value: _19.333__

d. If the value of each fractional Xi in the solution of the LP relaxation is truncated to 0, a feasible solution is obtained, with objective value _11____.

Suppose that we wish to use branch-and-bound to solve this problem:



e. What restrictions on the original problem should be used to obtain subproblem #1 and subproblem #2?
Set variable with fractional value (namely X_3) equal to 0 and 1, respectively

f. Find the value of the LP relaxation of subproblem 1: _18.5 (if X_3=0)
Truncating fractional values gives objective: ___17____

g. Find the value of the LP relaxation of subproblem 2: _19.2_ (if X_3=1)
Truncating fractional values gives objective: ___12____

h. At this point, what are the tightest upper and lower bounds that you can state for the optimal value of the original problem: UB= _19.2__, LB= __17__

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B4. ***Benders' Decomposition of Capacitated Plant Location Problem***: Consider the problem of determining which one or more of four possible plants should be built in order to serve 6 customers at minimum cost. (Four of the plant sites are adjacent to customer locations.) The data are:

	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5	Customer 6	Plant Capacity	Fixed cost
Plant 1	0	17	77	43	93	52	10	8544
Plant 2	17	0	61	40	76	36	14	4050
Plant 3	77	61	0	60	30	39	15	1917
Plant 4	43	40	60	0	87	61	11	396
Demand	2	2	10	1	10	4		

```
(Total demand= 29)
```

Benders' decomposition is used to solve this problem, using the variation in which the master problem is not optimized-- instead a solution, if any, is found which is better than the incumbent).

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We begin by solving the subproblem with the trial set of plants {1,2,3,4}, i.e., build all four plants:

```
                Solution of
           Transportation Problem

Plants opened: # 1 2 3 4
```

```
Minimum transport cost     =   674
Fixed cost of plants       = 14907
Total                      = 15581
*** New incumbent!
```

```
                              Optimal Shipments
                              f|     to
                              r|
                              o|1 2  3 4 5 6  7
                              m|
                              1|2 0  0 0 0 0  8
                              2|0 2  0 0 5 4  3
                              3|0 0 10 0 5 0  0
                              4|0 0  0 1 0 0 10
```

(Demand pt #7 is dummy demand for excess capacity.)

```
                             __________________
                            |                  |
                            |  Dual Solution   |
                            |of Transportation |
                            |     Problem      |
                            |__________________|
```

Supply constraints

```
                              i=      1  2 3  4
                              U[i]=  46 46 0 46
```

Demand constraints

```
                         j=      1   2 3   4  5   6
                         V[j]=  ¯46 ¯46 0 ¯46 30 ¯10
```

Generated support is $\alpha Y+\beta$,
where $\beta = ^{-}936$, and

i	α[i]
1	9004
2	$\boxed{}$ ← missing value = $\boxed{4694}$ $= F_2 + U_2 \times S_2 = 4050 + 46 \times 14$
3	1917
4	902

This is support # 1
<><><><><><><><><><><><><><><><><><><><><><><><><><><><><><>

Next we solve the Master problem to get a new trial set of plants to be built:

Master Problem

Initial status vector: J= ϕ *(empty)*

(suboptimized, i.e., a solution Y such that v(Y) < incumbent.)

Trial set of plants: 2 3
with estimated cost 5675 < incumbent (= 15581)

Current status vectors for Balas' additive algorithm: J = {3, 2, -1, -4}

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Next the 2^{nd} subproblem is solved using the new trial set of plants (2 & 3):

Plants opened: # 2 3

```
Minimum transport cost     =  748
Fixed cost of plants       = 5967
```

```
 Total                     = 6715
*** New incumbent! ***  (replaces 15581)

Optimal Shipments
                               f|     to
                               r|
                               o|1 2  3 4 5 6 7
                               m|
                               2|2 2  0 1 5 4 0
                               3|0 0 10 0 5 0 0

(Demand pt #7 is dummy demand for excess capacity.)
                              _________________
                             |                 |
                             |  Dual Solution  |
                             |of Transportation|
                             |     Problem     |
                             |_________________|

Supply constraints

                                i=     1  2 3 4
                                U[i]=  0 46 0 0
Demand constraints

                           j=     1   2   3 4  5  6
                           V[j]=  0 -29 -46 0 -6 30

Generated support is αY+β,
where β= 104, and
                                    i α[i]

                                    1 8544
                                    2 4694
                                    3 1917
                                    4  396
This is support # 2
<><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><>
```

The master problem is solved again:

```
                                 Master Problem

Initial status vector: J = {3, 2, -1, -4}

*** No solution with v(Y) less than incumbent! ***
(Current incumbent: 6715, with plants #2 3 open)
```

a. What is the missing value of α_2 in the first support that is generated? 4694

b. Using the two supports that have been generated, what cost is estimated for the solution Y= (1,0,1,1), i.e., building plants 1, 3, & 4? Max {10887, 10961} = 10961
(Maximum value of the two supports evaluated at Y=1,0,1,1)

c. Which, if any, of the subproblem (transportation) problem solutions are degenerate? *None: in each of the two transportation problems, the number of positive shipments = #sources + #destinations - 1*

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B5. Lagrangian Relaxation: Consider the following (capacitated) plant location problem: Three plant locations are considered. The annual capacity S_i of each plant i (if built) is specified, together with the annual cost F_i of the capital investment. Also shown in the table below are the annual demands D_j of four customer markets (j=A,B,C,D) which are to be supplied by these plants, together with the cost of shipments between each plant-customer pair:

	Customer A	Customer B	Customer C	Customer D	Capacity	Fixed cost
Plant 1	2	3	5	4	9	60
Plant 2	1	4	6	5	10	75
Plant 3	4	5	2	1	11	80
Demand	5	7	4	3		

We formulate the problem as a mixed-integer LP problem, with variables

Y_i = 1 if plant i is built; otherwise 0

X_{ij} = annual shipment from plant i to customer j

$$\text{Min} \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}X_{ij} + \sum_{i=1}^{3} F_i Y_i$$
$$\text{s.t.}$$
$$\sum_{j=1}^{4} X_{ij} \le S_i Y_i,\ i = 1,2,3$$
$$\sum_{i=1}^{3} X_{ij} = D_j,\ j = 1,2,3,4$$
$$X_{ij} \ge 0,\ Y_i \in \{0.1\},\ \forall i\&j$$

a. Apply Lagrangian relaxation to the *plant capacity* constraints of your formulation. Write the Lagrangian subproblem. with non-negative Lagrangian variable λ_i for each plant:

$$\text{Min} \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}X_{ij} + \sum_{i=1}^{3} F_i Y_i + \sum_{i=1}^{3} \lambda_i\left(\sum_{j=1}^{4} X_{ij} - S_i Y_i\right)$$
$$\text{s.t.} \sum_{i=1}^{3} X_{ij} \ge D_j \text{ for } j = 1,2,3,4$$
$$X_{ij} \ge 0,\ Y_i \in \{0,1\} \text{ for all } i\&j$$

or,

$$\text{Min} \sum_{i=1}^{3}\sum_{j=1}^{4} \left(C_{ij} + \lambda_i\right)X_{ij} + \sum_{i=1}^{3} \left(F_i - \lambda_i S_i\right)Y_i$$
$$\text{s.t.} \sum_{i=1}^{3} X_{ij} \ge D_j \text{ for } j = 1,2,3,4$$
$$X_{ij} \ge 0,\ Y_i \in \{0,1\} \text{ for all } i\&j$$

b. Let the Lagrangian multipliers be λ= (4,7,5). Solve the Lagrangian relaxation.

Compute the cost coefficients of the Lagrangian relaxation

	$C_{i1} + \lambda i$	$C_{i2} + \lambda i$	$C_{i3} + \lambda i$	$C_{i4} + \lambda i$	Fi - λi Si
Plant 1	**2+4=6**	**3+4=7**	5+4=9	4+4=8	60–4×9= +24
Plant 2	1+7=8	4+7=11	6+7=13	5+7=12	75–7×10= +5
Plant 3	4+5=9	5+5=10	**2+5=7**	**1+5=6**	80–5×11= +25

The optimal solution is $X_{11} = X_{12} = X_{33} = X_{34} = 1$, $Y_1=Y_2=Y_3=0$ with objective value 6+7+7+6=26

c. What is the objective value of the Lagrangian relaxation? _____

d. Is the objective value of the Lagrangian relaxation a *lower* bound or *upper* bound on the optimum of the original problem? _________

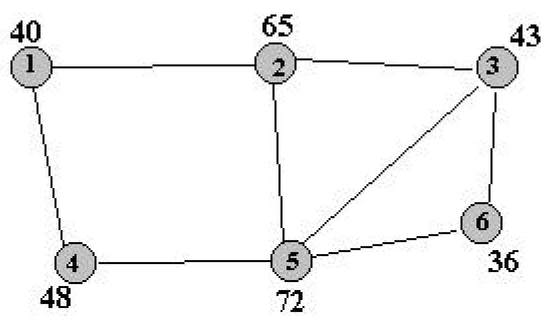
e. For each of the Lagrangian multipliers, specify whether it should be adjusted upward or downward in order to improve the bound.

Lagrangian multiplier λ	Direction of movement (up, down, or unchanged?)
1	
2	
3	

❍ ❍ ❍ ❍ ❍ ❍ ❍ **PART C** ❍ ❍ ❍ ❍ ❍ ❍

Integer programming model formulation: Be sure to define your variables!!!!

C1. Traffic monitoring device location. The following map shows the 8 intersections at which automatic traffic monitoring devices might be installed:



A station at any particular intersection can monitor all the road links meeting that intersection. Numbers next to nodes reflect the monthly cost (in thousands of dollars) of operating a station at that location.

At which nodes should stations be installed to provide full coverage at minimum total cost?

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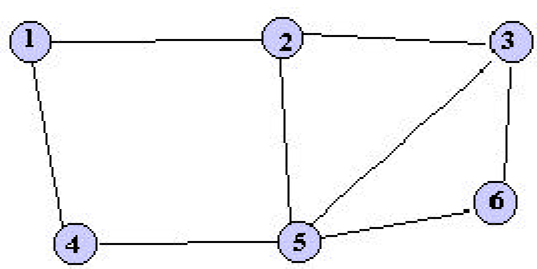
C2. Mailbox Location



The post office wishes to place the minimum number of mailboxes at intersections in the street network above in such a way that there is a mailbox at an intersection of every street in the network (allowing for the possibility that some streets may have mailboxes at both ends.)

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C3. Cassette Tape Allocation. You have been assigned to arrange the songs on the cassette version of Madonna's latest album. A cassette tape has two sides (side A and side B). The songs on each side of the cassette must total between 14 and 16 minutes in length. The length and type of each song are given in the table below:

Song	Type	Length (minutes)
1	Ballad	4
2	Hit	5
3	Ballad	3
4	Hit	2
5	Ballad	4
6	Hit	3
7	--	5
8	Ballad & Hit	4

The assignment of songs to the tape must satisfy the following four conditions:

i) Each side must have exactly two ballads.
ii) Side A must have at least 3 hit songs.
iii) Either song 5 or song 6 must be on side A.
iv) If songs 2 and 4 are both on side A, then song 5 must be on side B.

Formulate an integer LP with binary variables to find an arrangement satisfying these restrictions.

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Integer programming model formulation: Be sure to define your variables!!!!

D1. Black Box Company (BBC) is considering five new box designs of different sizes to package four upcoming lines of computer monitors. The following table shows the wasted space that each box would have if used to package each monitor. Missing values indicate a box that cannot be used for a particular monitor.

	Monitor			
Box	1	2	3	4
1	5	--	10	--
2	20	--	--	25
3	40	--	40	30
4	--	10	70	--
5	--	40	80	--

BBC wants to choose the smallest number of box designs needed to pack all products and to decide which box design to use for each monitor to minimize wasted space.

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D2. Dandy Diesel Mfg Co. assembles diesel engines for heavy construction equipment. Over the next 4 quarters the company expects to ship 40, 20, 60, and 15 units, respectively, but no more than 50 can be assembled in any quarter. There is a fixed cost of $2000 each quarter the line is set up for production, plus $200 per unit assembled. Engines may be held over in inventory at the plant for $100 per unit per month. Dandy seeks a minimum total cost production plan for the four quarters, assuming that there is no beginning or ending inventory.

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D3. Top T-shirt Company imprints T-shirts with cartoons & celebrity photographs. For each of their 4 pending contracts, the following table shows the number of days of production required, the earliest day the production can begin, and the day the order is due:

	Job 1	Job 2	Job 3	Job 4
Production time (days)	10	3	16	8
Earliest start	0	20	1	12
Due date	12	30	20	21

The company wants a schedule which will minimize the sum of tardiness (lateness) of the jobs.
Define

T_i = starting time for job i

L_i = lateness (tardiness) for job i

Define any additional variables that you wish to use, and formulate an integer LP

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