LP Sensitivity Analysis

Zales Jewelers uses rubies and sapphires to produce two types of rings. A type 1 ring requires 2 rubies, 3 sapphires, and 1 hour of jeweler's labor. A type 2 ring requires 3 rubies, 2 sapphires, and 2 hours of jeweler's labor. Each type 1 ring sells for $400, and each type 2 ring sells for $500. All rings produced by Zales can be sold. At present, Zales has 100 rubies, 120 sapphires, and 70 hours of jeweler's labor available. Extra rubies can be purchased at a cost of $100 each. Market demand requires that the company produce at least 20 type 1 rings and at least 25 type 2 rings. To maximize profit, Zales should solve the following LP:

X1= type 1 rings produced.
X2= type 2 rings produced
R = number of rubies purchased.

$$
\begin{aligned}
\text{MAX } & z=400X_1 + 500X_2 - 100R \\
\text{s.t. } & 2X_1 + 3X_2 - R \le 100 \\
& 3X_1 + 2X_2 \le 120 \\
& X_1 + 2X_2 \le 70 \\
& X_1 \ge 20 \\
& X_2 \ge 25 \\
& X_1 \ge 0,\ X_2 \ge 0
\end{aligned}
$$

The LINDO output for this problem follows:

```
MAX     400 X1 + 500 X2 - 100 R
SUBJECT TO
       2)   2 X1 + 3 X2 - R <=   100
       3)   3 X1 + 2 X2 <=   120
       4)   X1 + 2 X2 <=   70
       5)   X1 >=   20
       6)   X2 >=   25
END
      OBJECTIVE FUNCTION VALUE
      1)      19000.00

VARIABLE        VALUE          REDUCED COST
      X1        20.000000           .000000
      X2        25.000000           .000000
       R        15.000000           .000000

     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)          .000000       100.000000
      3)        10.000000          .000000
      4)          .000000       200.000000
      5)          .000000          .000000
      6)         -.000000      -200.000000
```

Reminder: According to LINDO's definitions,

- "`REDUCED COST`" is the rate by which the optimal value of the objective function *deteriorates* as a nonbasic variable is increased.
  (Therefore, units of `REDUCED COST` are $[\text{units of objective}]/[\text{units of variable}]$, e.g., \$/ring)

- "`DUAL PRICE`" is the rate by which the optimal value of the objective function *improves* as a right-hand-side is increased. (Therefore, units of `DUAL PRICE` are $[\text{units of objective}]/[\text{units of RHS}]$, e.g., \$/stone)

```
RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
       X1      400.000000         INFINITY       100.000000
       X2      500.000000       200.000000         INFINITY
        R     -100.000000       100.000000       100.000000

                           RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE        ALLOWABLE
                    RHS          INCREASE         DECREASE
        2      100.000000        15.000000         INFINITY
        3      120.000000         INFINITY        10.000000
        4       70.000000         3.333333          .000000
        5       20.000000          .000000         INFINITY
        6       25.000000          .000000         2.500000

  THE TABLEAU
ROW  (BASIS)    X1     X2     R      SLK 2 SLK 3   SLK 4 SLK 5  SLK 6
1    ART       .000   .000   .000 100.000   .000 200.000   .000 200.000 19000.000
2       X2     .000 1.000   .000     .000  .000     .000  .000   -1.000    25.000
3    SLK 3     .000   .000   .000     .000 1.000  -3.000  .000   -4.000    10.000
4        R     .000   .000 1.000   -1.000  .000    2.000  .000    1.000    15.000
5       X1   1.000   .000   .000     .000  .000    1.000  .000    2.000    20.000
6    SLK 5     .000   .000   .000     .000  .000    1.000 1.000   2.000      .000
```

Use the LINDO output to answer the following questions *(wherever possible)*:

a. Suppose that, instead of $100, each ruby costs $190. Should Zales still purchase rubies?

b. Suppose that Zales were required to produce at least 23 *(not 25)* Type 2 rings. What would Zales' profit now be? ________

c. What is the most that Zales should be willing to pay for another hour of jeweler's time? _______
How much should they be willing to pay for another *four* hours? _______

d. Suppose that another 3 hours of jeweler's time became available. By using the "substitution rates" in the tableau, explain what changes would result in the number of rings of each type which would be produced, as well as the number of additional rubies (if any) which would be purchased.

e. What is the most that Zales should be willing to pay for another sapphire? _____

f. Zales is considering producing <u>Type 3</u> rings. Each type 3 ring can be sold for $550 and requires 4 rubies, 2 sapphires, and 1 hour of jeweler's labor. Without re-running the LP, can you determine whether Zales should produce any type 3 rings? ______
(You need not determine how many, if any, should be produced.)