Team# ___ (names: _____)

56:171 Operations Research
Lab/Discussion Exercises: Transportation & Assignment Problems
Week of October 1-3, 2002

Part 1. Transportation Simplex Method

Consider the basic feasible solution of the transportation below:

dstn→ ↓source	1	2	3	4	5	Supply
A	| 12	| 8 **7**	| 9	| 10	| 11 **2**	9
B	| 10 **2**	| 11	| 12	| 11 **5**	| 14	7
C	| 9 **2**	| 7	| 11 **2**	| 14	| 8	4
D	| 13	| 12	| 13 **3**	| 12	| 12 **4**	7
E	| 8	| 9	| 10	| 9	| 10 **3**	3
Demand=	4	7	5	5	9	

a. Does this solution have a full set of basic variables? (If not, select additional variable(s) to be basic.)

b. Write a set of values for the 10 dual variables (5 for sources, 5 for destinations) above and on the left, in the box with the respective source or destination number, starting with $U_A = 0$.

c. Compute the reduced costs of the nonbasic variables X_{E1} & X_{B5}. ($\underline{C}_{E1}$ = _____ & $\underline{C}_{B5}$ = _____)

d. Starting by fixing $V_3 = 0$, find a different set of values for the 10 dual variables. Recompute the reduced costs of X_{E1} & X_{B5}. ($\underline{C}_{E1}$ = _____ & $\underline{C}_{B5}$ = _____)

Consider now the transportation problem with the *degenerate* basic solution shown below. Seven shipments are indicated, so two additional variables must be basic, even though zero! *Note that, for example, X_{B2} cannot be in the basis, because that would create a "cycle" $X_{B2} \rightarrow_{B5} \rightarrow X_{A5} \rightarrow X_{A2} \rightarrow X_{B2}$ which cannot occur in a basic solution.*

dstn→ ↓source	1	2	3	4	5	Supply
A	| 12	| 8 **7**	| 9	| 10	| 11 **2**	9
B	| 10	| 11	| 12	| 11	| 14 **7**	7
C	| 9 **4**	| 7	| 11	| 14	| 8	4
D	| 13	| 12	| 13 **2**	| 12 **5**	| 12	7
E	| 8	| 9	| 10 **3**	| 9	| 10	3
Demand=	4	7	5	5	9	

e. Suppose that X_{A1} is the eighth basic variable. What other variable can included as the *ninth* basic variable? _____)

f. Compute a new set of values of the dual variables, with U_A=0. (If your choice of ninth basic variable is invalid, you will not be able to determine all of the dual variables.)

g. Suppose X_{E1} enters the basis. Which variable leaves the basis?

Consider the transportation problem below:

dstn→ ↓source	1	2	3	4	5	Supply
A	| 12	| 5 **7**	| 9	| 10	| 9 **2**	9
B	| 7 **2**	| 11	| 12	| 11 **5**	| 14	7
C	| 9 **2**	| 13	| 11 **2**	| 14	| 15	4
D	| 8	| 12	| 8 **3**	| 12	| 12 **4**	7
E	| 8	| 9	| 10	| 9	| 10 **3**	3
Demand=	4	7	5	5	9	

h. Compute a set of values of the dual variables, and confirm that the solution shown is optimal.

i. There is another basic optimal solution. How do the values of the variables differ from that shown above?

Part 2: Assignment Problem

Consider the problem of assigning machines (rows) to jobs (columns), with cost matrix

|	1	2	3	4	5
1	7	7	7	8	7
2	10	2	6	12	12
3	14	9	5	5	11
4	1	12	15	7	5
5	3	12	11	6	11

a. Perform row reduction so that each row contains a zero, and write the new matrix below:

	1	2	3	4	5
1					
2					
3					
4					
5					

Note that column reduction isn't required in this case.

b. Find the smallest number of lines required to cover all the zeroes. _____

c. Reduce the matrix again, so that a zero will appear that is not covered by the lines you selected above:

	1	2	3	4	5
1					
2					
3					
4					
5					

d. How many lines are required to cover all the zeroes now? _____ *(You need not proceed to complete solving the problem.)*

Consider the reduced assignment problem shown below:

|	1	2	3	4	5
1	0	0	3	1	3
2	1	2	0	2	1
3	4	3	5	5	0
4	1	0	5	3	5
5	0	2	1	0	11

e. How many lines are required to cover all of the zeroes? _____

f. Find a zero-cost solution of the problem with this cost matrix.

Answer: assign

machine 1→ ___ , machine 2→ ___

machine 3→ ___ , machine 4→ ___ ,

machine 5→ ___