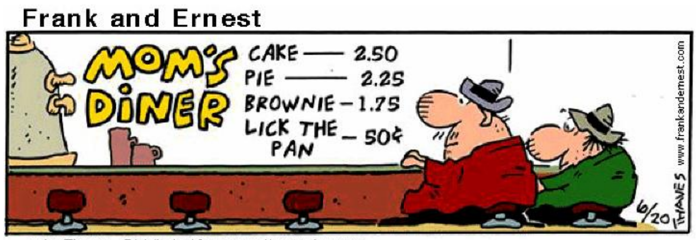
A LINGO model of a staffing problem

Mama's Kitchen



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"Mama's Kitchen" serves from 5:30 a.m. each morning until 1:30 p.m. in the afternoon.

Tables are set and cleared by busers working **4-hour shifts** beginning on the hour from 5:00 a.m. (shift #1) through 10:00 a.m. (shift #6).

Most are college students who hate to get up in the morning, so Mama's pays $**9** per hour for the 5:00, 6:00, and 7:00 a.m. shifts, and $**7.50** per hour for the others.

The manager seeks a **minimum cost staffing plan** that will have at least a minimum number of busers on duty each hour:

	5 am	6 am	7 am	8 am	9 am	10am	11am	Noon	1 pm
#reqd	**2**	**3**	**5**	**5**	**3**	**2**	**4**	**6**	**3**

Mama's Kitchen

Decision variables

Xi = the # of employees who start to work on i[th] shift. (i = 1, 2, ... , 6)

LP Formulation

```
MIN 36 X1 + 36 X2 + 36 X3 + 30 X4 + 30 X5 + 30 X6
SUBJECT TO
 X1                           >=   2  (min  # of busers on duty at  5am)
 X1 + X2                      >=   3  (min  # of busers on duty at  6am)
 X1 + X2 + X3                 >=   5  (min # of busers on duty at  7am)
 X1 + X2 + X3 + X4            >=   5  (min # of busers on duty at  8am)
      X2 + X3 + X4 + X5       >=   3  (min # of busers on duty at  9am)
           X3 + X4 + X5 + X6 >=   2  (min # of busers on duty at 10am)
                X4 + X5 + X6 >=   4  (min # of busers on duty at 11am)
                     X5 + X6 >=   6  (min # of busers on duty at 12pm)
                          X6 >=   3  (min # of busers on duty at  1pm)
 Xi >= 0  (for i = 1,2,3,4,5,6)        (Sign restrictions)
```

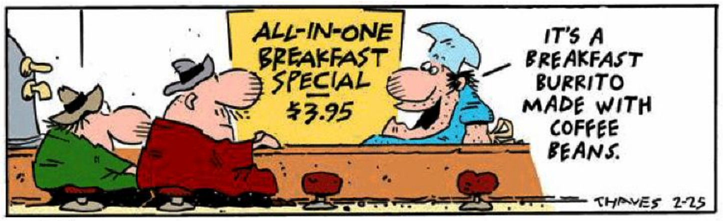


```
OBJECTIVE FUNCTION VALUE
        1)      360.0000

  VARIABLE        VALUE          REDUCED COST
        X1         3.000000          0.000000
        X2         0.000000          0.000000
        X3         2.000000          0.000000
        X4         0.000000          0.000000
        X5         3.000000          0.000000
        X6         3.000000          0.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         1.000000          0.000000
        3)         0.000000          0.000000
        4)         0.000000         -6.000000
        5)         0.000000        -30.000000
        6)         2.000000          0.000000
        7)         6.000000          0.000000
        8)         2.000000          0.000000
        9)         0.000000        -30.000000
       10)         0.000000          0.000000
```

Use a modeling language (e.g. LINGO or MPL) to formulate this LP!



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Number the hours j =1, …,9 where #1=5am, …, #9=1pm.

The shifts which are on-duty in hour #*i* are therefore

$i,\ i-1,\ i-2,\ \&\ \ i-3,$

where we omit shifts numbered less than # 1.

The mathematical statement of the problem:

$$\text{Minimize } \sum_{j=1}^{6} C_j X_j$$

subject to

$$\sum_{\substack{j=i-3 \\ j\geq 1}}^{i} X_j \geq R_i, \quad i=1,\ldots 9$$

$$X_j \geq 0, \quad j=1,\ldots 6$$

LINGO model with sets

```
MODEL:  ! Mama's Kitchen;

SETS:
   HOUR /1..9/: RQMT;
   SHIFT /1..6/: COST, X;
ENDSETS

DATA:
   RQMT =   2  3  5  5  3  2  4  6  3;
   COST =  36 36 36 30 30 30;
ENDDATA

MIN = @SUM(SHIFT:  COST*X);

@FOR(HOUR(I):
   @SUM(SHIFT(J)| J #GE#1 #AND# J #GE# I-3 #AND# J #LE# I:
                    X(J)) >= RQMT(I);
   );

END
```


The solution—*same cost ($360) as before, but different staffing plan!*

Variable	Value	Reduced Cost
X(1)	5.000000	0.0000000
X(2)	0.0000000	0.0000000
X(3)	0.0000000	0.0000000
X(4)	0.0000000	30.00000
X(5)	3.000000	0.0000000
X(6)	3.000000	0.0000000