Sensitivity Analysis in LP.

During each 6-hour period of the day, the Bloomington Police Department needs at least the number of police officers shown below:

#	Time Period	# of Officers Required
1	12 midnight- 6 a.m.	12
2	6 a.m. - 12 noon	8
3	12 noon - 6 p.m.	6
4	6 p.m. - 12 midnight	15

Officers can be hired to work either 12 consecutive hours or 18 consecutive hours. Officers are paid $12 per hour for each of the first 12 hours a day they work, and are paid $18 per hour for each of the next 6 hours they work in a day. The city wishes to minimize the cost of meeting Bloomington's daily police requirements.

Number the four 6-hour time intervals by integers 1,2,3, & 4. Define decision variables

X_t = number of officers assigned to 12-hour shift beginning in time interval t (t=1,2,3,4)

Y_t = number of officers assigned to 18-hour shift beginning in time interval t (t=1,2,3,4)

Thus, for example, Y_3 is the number of officers who begin their shift at noon and complete their shift at 6 a.m. the following morning.

Ignoring the restriction that the number of officers for each shift must be integer-valued and solving the problem as an ordinary LP, LINDO gives us the following solution, which happens to satisfy the integer restrictions:

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MIN  144 X1 + 144 X2 + 144 X3 + 144 X4 + 252 Y1 + 252 Y2 + 252 Y3 + 252 Y4
  SUBJECT TO
         2)   X1 + X4 + Y1 + Y3 + Y4 >=   12
         3)   X1 + X2 + Y1 + Y2 + Y4 >=   8
         4)   X2 + X3 + Y1 + Y2 + Y3 >=   6
         5)   X3 + X4 + Y2 + Y3 + Y4 >=   15
  END
        OBJECTIVE FUNCTION VALUE
        1)      3132.000

  VARIABLE        VALUE          REDUCED COST
        X1         3.000000           .000000
        X2          .000000           .000000
        X3         1.000000           .000000
        X4         9.000000           .000000
        Y1          .000000         72.000000
        Y2         5.000000           .000000
        Y3          .000000         72.000000
        Y4          .000000           .000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         -.000000        -36.000000
        3)         -.000000       -108.000000
        4)         -.000000        -36.000000
        5)         -.000000       -108.000000
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RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	144.000000	.000000	36.000000
X2	144.000000	INFINITY	.000000
X3	144.000000	.000000	.000000
X4	144.000000	.000000	.000000
Y1	252.000000	INFINITY	72.000000
Y2	252.000000	.000000	72.000000
Y3	252.000000	INFINITY	72.000000
Y4	252.000000	INFINITY	.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	12.000000	5.000000	1.000000
3	8.000000	1.000000	5.000000
4	6.000000	5.000000	.500000
5	15.000000	1.000000	5.000000

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ROW	(BASIS)	X1	X2	X3	X4	Y1	Y2
1	ART	.000	.000	.000	.000	72.000	.000
2	X1	1.000	1.000	.000	.000	2.000	.000
3	X4	.000	-1.000	.000	1.000	-1.000	.000
4	Y2	.000	.000	.000	.000	-1.000	1.000
5	X3	.000	1.000	1.000	.000	2.000	.000

ROW	Y3	Y4	SLK 2	SLK 3	SLK 4	SLK 5	
1	72.000	.000	36.000	108.000	36.000	108.000	-3132.000
2	1.000	.000	-1.000	.000	-1.000	1.000	3.000
3	.000	1.000	.000	.000	1.000	-1.000	9.000
4	-1.000	1.000	1.000	-1.000	1.000	-1.000	5.000
5	2.000	-1.000	-1.000	1.000	-2.000	1.000	1.000

___1. If the officers who work a 12-hour shift from midnight to noon were to receive a $1/hour raise, the number assigned to this shift would

a. increase c. remain the same e. *NOTA*
b. decrease d. insufficient info. given

___2. If <u>all</u> officers who work an 18-hour shift were to receive a $1/hour raise, the number of officers working these shifts would

a. increase c. remain the same e. *NOTA*
b. decrease d. insufficient info. given

___3. The LP problem above has

a. exactly one optimal sol'n c. multiple solutions e. insufficient info. given
b. a degenerate solution d. no optimal solution f. *NOTA*

4. Reducing the requirement for officers working during the midnight-6 a.m. period by two (from the current 12 to 10) would lower the total cost by _____

5. Increasing the requirement for officers working during the midnight-6 a.m. period by two (from the current 12 to 14) would increase the total cost by _____

___6. Increasing the requirement for officers working during the midnight-6 a.m. period by two (from the current 12 to 14) would be equivalent to

a. increasing *slack* variable SLK_2 by two units.
b. increasing *surplus* variable SLK_2 by two units.
c. decreasing *slack* variable SLK_2 by two units..
d. decreasing *surplus* variable SLK_2 by two units.
e. *NOTA*

7. Using substitution rates and your answer in (6), determine the change in the basic variables resulting from an increase of one person in the requirement during the midnight-6 a.m. period:

	increase	decrease	amount
X1	[__]	[__]	_____
X2	[__]	[__]	_____
X4	[__]	[__]	_____
Y2	[__]	[__]	_____

8. Suppose that it were required that one officer be assigned an 18-hour shift from noon to 6 a.m. The effect on the total cost would be

	increase	decrease	amount
Cost	[__]	[__]	by _____

The effect on the variables in the current solution would be

	increase	decrease	amount
X1	[__]	[__]	by _____
X2	[__]	[__]	by _____
X4	[__]	[__]	by _____
Y2	[__]	[__]	by _____