Process Selection/Product Mix Problem

(from *Optimization Modeling with LINGO*, by Linus Schrage, LINDO Systems, Inc., page 98)

The American Metal Fabricating Company (AMFC) produces various products from steel bars. One of the initial steps is a shaping operation performed by rolling machines. There are three machines available for this purpose, the B3, B4, and B5. The following table gives their features:

Machine	Speed (ft/min.)	Allowable raw material thickness (in.)	Available hrs/week	Labor cost ($/hour)
3	150	3/16 to 3/8	35	$10
4	100	5/16 to ½	35	$15
5	75	3/8 to 3/4	35	$17

(That is, machines that process larger material operate at slower speed.)

This week there are three products that must be produced. AMFC must produce at least 218,000 feet of ¼" material, 114,000 feet of $^3/_8$" material, and 111,000 feet of ½" material. The profit contributions ($/foot) excluding labor for these three products are 0.017, 0.019, and 0.02, respectively. These prices apply to all production (e.g., including any in excess of the required production). The shipping department has a capacity limit of 600,000 feet per week, regardless of the thickness.

Decision variables:

X_{34} = 1000's of ft of ¼" produced on machine 3
X_{38} = 1000's of ft of $^3/_8$" produced on machine 3
X_{48} = 1000's of ft of $^3/_8$" produced on machine 4
X_{58} = 1000's of ft of $^3/_8$" produced on machine 5
X_{42} = 1000's of ft of ½" produced on machine 4
X_{52} = 1000's of ft of ½" produced on machine 5

For the objective function, we must have the profit contribution including labor costs. When this is done, we obtain

Variable	Profit contribution ($/ft)
X_{34}	0.01589
X_{38}	0.01789
X_{48}	0.01650
X_{58}	0.01522
X_{42}	0.01750
X_{52}	0.01622

Clearly, there will be four constraints corresponding to AMFC's three scarce machine resources and its shipping department capacity. There should be three more constraints due to the production requirements in the three products.

For the machine capacity constraints, we want the number of hours required for 1000 feet processed. The speed of machine 3 is 150 ft/min $\times$ 60 min/hr = 9000 ft/hr so that processing 1000 ft requires 1/9 = 0.11111 hr. Similar figures for machines 4 and 5 are 0.16667 hrs/1000ft and 0.22222 hrs/1000ft.

LP Model:

$$\text{Maximize} \quad \sum_{(i,j)} P_{ij} X_{ij}$$

$$\text{subject to} \quad \sum_{(i,j)} X_{ij} \le K_s$$

$$\sum_{j \in thickness} T_i X_{ij} \le A_i \quad \text{for each } i \in machine$$

where A_3 is available hrs/wk on machine 3, … K_S = shipping capacity (Kft/week)

$$\sum_{i \in machine} X_{ij} \ge R_j \quad \text{for all } j \in thickness$$

$$X_{ij} \ge 0 \quad \text{for all } i \,\&\, j$$

LINDO model

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MIN 15.89 X34 + 17.89 X38 + 16.5 X48 + 17.5 X42 + 15.22 X58
      + 16.22 X52
ST
      X34 + X38 + X48 + X42 + X58 + X52 <=   600
      0.111111 X34 + 0.111111 X38 <=   35
      0.166667 X48 + 0.166667 X42 <=   35
      0.222222 X58 + 0.222222 X52 <=   35
      X34 >=   218
      X38 + X48 + X58 >=   114
      X42 + X52 >=   111
END
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Partial LINGO Model (file AMFC.lg4):

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MODEL:

SETS:
   MACHINE / B3, B4, B5 / : A, T;
! A is the available hours per week,
  T is hours required per thousand feet of throughput ;
   THICKNESS / FOURTH, EIGHT, HALF / : R;
! R is the amount of each thickness required to be produced;
   METHOD ( MACHINE, THICKNESS ) : X, P, B;
!X is the variable,
 P the objective (profit) coefficients, and
 B is a Boolean indicating if it is possible to produce the given thickness;
ENDSETS

DATA:
!  Hours/week available on each machine;
      A =   35,   35,   35;
! Hours per 1000 feet for each machine;
      T = .11111 .16667 .22222;
! Amount needed of each product;
      R = 218    114    111;
! Profit by product and machine;
      P = 15.89, 17.89,  0,
           0,    16.5,  17.5,
           0,    15.22, 16.22;
! Which products can be made on which machine;
  B =      1,     1,     0,
           0,     1,     1,
           0,     1,     1;
! Shipping capacity per day;
  KS = 600;
ENDDATA

!  objective & constraints go here ;

END
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a. Load LINGO, then download & read the AMFC.lg4 file.
b. "Translate" the mathematical statement of the LP into LINGO statements, and solve the problem.
c. E-mail the revised AMFC.lg4 file to either dbricker or hsohn before deleting it.