What is reliability?

Reliability Definition

Classical Definition
Probability that a component, activity, resource, process, and so on is operational.

Expanded Definition
Probability that a certain performance measure, e.g., project due date, cost, is realized.
Example: Job Interview Selection

Three interview schemes

A

R = .8 Interview success rate

R \_1

R _2

$60k

B

R \_1

R _2

$55k

Up to two job interviews are required for each job

C

R \_1

R _2

$50k

Question

- How to determine reliability of a process represented with an IDEF3 model?

Question

- Can systems engineering methods be used to determine the process reliability?
Serial Activities

\[ R_S = \prod_{j=1}^{n} R_j \]

**EXAMPLE**

For \( R_1 = .8 \) and \( R_2 = .9 \)
\[ R_S = .8 \times .9 = .72 \]

AND Junction Activities

\[ R_S = \prod_{j=1}^{n} R_j \]

Is equivalent to

Exclusive OR Activities

\[ R_S = \sum_{j=1}^{n} p_j R_j \]

where:
- \( R_S \) is the system reliability,
- \( p_j \) is an occurrence probability of activity \( j \), with \( \sum_{j=1}^{n} p_j = 1 \)
EXAMPLE

$$P_1 = .7, P_2 = .3, \text{ where } P_1 + P_2 = 1$$

$$R_1 = .8, R_2 = .9$$

$$R_s = .7 \times .8 + .3 \times .9 = .83$$

$$R_s = \sum_{j=1}^{n} P_j R_j$$

OR Junction Activities

(Probability that at least k out of n elements are operational)

EXAMPLE

For $k = 1$ and $n = 2$

$$R_s = R_1 + R_2 - R_1 R_2 = .8 + .9 - .72 = .98$$

$$R_s = P\left[\sum_{1}^{n} x_j \geq k\right]$$

For $R_1 = .8$ and $R_2 = .9$

$$Rs = .8 \times .9 = .72$$

For $P_1 = .7, P_2 = .3$

$$R_1 = .8, R_2 = .9$$

$$Rs = .7 \times .8 + .3 \times .9 = .83$$

For $k = 1$ and $n = 2$

$$Rs = R_1 + R_2 - R_1 R_2 = .8 + .9 - .72 = .98$$

$60k$

$55k$

$50k$
Reliability Computation

- Reduction Method
- Path Set Method
- Cut Set Method

The basic idea behind the reduction approach of an IDEF3 model is to reduce its size by combining appropriate parallel AND, OR, and exclusive OR branches, and the activities until a single equivalent activity is obtained. This equivalent activity represents the reliability of the original IDEF3 model.

EXAMPLE: Steps of the Reduction Algorithm

**EXAMPLE 1**

R24 = R2 R4

**EXAMPLE 2**

R67 = R6 R7

R24 = R2 R4
\[ R_{24} = R_2 R_4 \]
\[ R_{67} = R_6 R_7 \]
\[ R_{567} = R_5 + R_{67} - R_5 R_{67} \]
\[ R_{3567} = R_3 R_{567} \]
\[ R_{234567} = P_{24} R_{24} + P_{3567} R_{3567} \] with \[ P_{24} + P_{3567} = 1 \]

Rs = R_{12345678} = R_1 R_{234567} R_8

Assuming \( k = 1 \)

\[
\begin{align*}
R_1 &= 0.99 \\
R_2 &= 0.98 \\
R_3 &= 0.97 \\
R_4 &= 0.99 \\
R_5 &= 0.95 \\
R_6 &= 0.97 \\
R_7 &= 0.95 \\
R_8 &= 0.96 \\
\end{align*}
\]

Case Study

\[
\begin{align*}
R_{67} &= R_5 + R_6 - R_5 R_6 = 0.9985 \\
R_{78} &= P_7 R_7 + P_8 R_8 = 0.957 \quad (P_7 = 0.3, P_8 = 0.7) \\
R_{56} &= R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 = 0.8725 \\
\end{align*}
\]

Name advantages and disadvantages of the reduction approach?

Other Approaches

- Path Set Algorithm
- Cut Set Algorithm

Activity Number and Name:
1. Define system requirements
2. Develop weight, size, and shape specifications
3. Perform detailed design of subsystem A
4. Perform detailed design of subsystem B
5. Design packaging module 1A
6. Design packaging module 1B
7. Design packaging module 2A
8. Design packaging module 2B
9. Prototype build
**Terminology**

**Start point**
Node in the network which has only outgoing arrows (O point).

**End point**
Node in the network which has only incoming arrows (D point).

**Path**
A path from the start (origin) point O to the end point D, which includes activities whose occurrence ensures the occurrence of the end point D.

**Cut set**
A set of activities whose failure ensures the failure of the end point D.

**Tree**
A representation that shows the relationship among all activities in the model from the start point O to the end point D.

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**Reliability Evaluation Algorithm (Path Set/Cut Set)**

Step 1. Representation of an IDEF3 process diagram as a reliability network.

Step 2. Transformation of the reliability network into a tree by the path tree algorithm.

Step 3. Generation of path sets and/or cut sets from the tree.

Step 4. Compute of the reliability measure of interest.

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**Representation of an IDEF3 Model as a Reliability Network**

**IDEF3 graphical components and corresponding reliability network**

<table>
<thead>
<tr>
<th>IDEF3 Graphical Components</th>
<th>Equivalent Reliability Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>![IDEF3 Component 1]</td>
<td>![Equivalent Reliability Network 1]</td>
</tr>
<tr>
<td>![IDEF3 Component 2]</td>
<td>![Equivalent Reliability Network 2]</td>
</tr>
<tr>
<td>![IDEF3 Component 3]</td>
<td>![Equivalent Reliability Network 3]</td>
</tr>
</tbody>
</table>
EXAMPLE 3

The path sets are: \{1, 2, 4, 8\}, \{1, 3, 5, 8\}, \{1, 3, 6, 7, 8\}.

Reliability of Path Sets

- $R_{P1} = (R_1 \times R_2 \times R_4 \times R_8)$
- $R_{P2} = (R_1 \times R_3 \times R_5 \times R_8)$
- $R_{P3} = (R_1 \times R_3 \times R_6 \times R_7 \times R_8)$

For all $R_i = .9$
- $R_{P1} = R_{P2} = .9^4$
- $R_{P3} = .9^5$
Path Set Method

Algorithm 2

Models with Cycles

Step 1: Begin with the start point O at the top of the tree (level k = 1).
Step 2: Place the activities connected to the start point O at the next level (level k = 2).
Step 3: Obtain level k + 1 activities connected to the activities at level k.
Step 4: If any activity obtained at level k + 1 is already included in a path, disregard this activity.
Step 5: If an activity obtained at level k + 1 is the same as any activity obtained at the higher level of any other path, fathom the path.
Step 6: If there are no more activities at the level k + 1, set k = k + 1 and repeat Steps 3 - 5 until each activity at the last level is fathomed or corresponds to the end point D.
Step 7: Formulate path sets.

EXAMPLE

Reliability network of the IDEF3 model

The path sets are: 

- {1, 2, 4, 8, 10},
- {1, 2, 4, 9, 10},
- {1, 3, 6, 7, 10},
- {1, 3, 5, 10},
- {1, 3, 8, 10},
- {1, 3, 8, 9, 4, 10}

Reliability of Path Sets

Product of elementary reliabilities

- {1, 2, 4, 8, 10},
- {1, 2, 4, 8, 9, 10},
- {1, 3, 6, 7, 10},
- {1, 3, 5, 10},
- {1, 3, 8, 10},
- {1, 3, 8, 9, 4, 10}
Discussion

Failure of R9 impacts the system as more than one path through R9 might be required.

Algorithm 3

**Generation of Cut Sets**

**Assumes that the path sets are available**

- **Step 0**: Initialize the current solution set $S_1 = \{\emptyset\}$, and two working sets $S_2 = \{\emptyset\}$ and $S_3 = \{\emptyset\}$.
- **Step 1**: Place activities represented by the start point O and end point D in set $S_1$.
- **Step 2**: Select a path set. If any activity in the path set selected occurs in each path set, add this activity to the set $S_1$.
- **Step 3**: Place the activities of the path set that are not in set $S_1$ in set $S_2$.
- **Step 4**: Place all the activities of the model that are not in set $S_1$ in set $S_3$, and set the number of activities in a q-tuple, $q = 2$.
- **Step 5**: Find all possible q-tuples of the activities in $S_2$ and $S_3$.
  - Each q-tuple is a cut set, if each path set is covered by an activity from this cut set.
  - Add the tuples obtained to set $S_1$. If the maximum number of activities in the cut sets generated $q = total\ number\ of\ path\ sets$ in the model, then repeat step 5 by creating q tuples from $S_2$ and $S_3$; otherwise, stop.

**Notation Clarification**

- $S_1 = \{O, D, + activities\ common\ to\ all\ path\ sets\}$
- $S_2 = \{\emptyset\}$
- $S_3 = \{Activities\ of\ all\ path\ sets\ (the\ model)\} - \{Activities\ of\ the\ selected\ path\}$

**EXAMPLE**

The path sets for the model in one of the previous examples:

- $\{1, 2, 4, 8\}$, $\{1, 3, 5, 8\}$, $\{1, 3, 6, 7, 8\}$
Steps of Algorithm 3

Path Sets:
{1, 2, 4, 8}, {1, 3, 5, 8}, {1, 3, 6, 7, 8}

Step 0: Initialize: S1 = {Ø}, S2 = {Ø}, S3 = {Ø}.
Step 1: Activities 1 and 8 are added to set S1. Therefore the current solution S1 = {1, 8}.
Step 2: The path {1, 2, 4, 8} is selected.
Step 3: Activities 2 and 4 are included in S2 = {2, 4}.
Step 4: {All activities of the model} – {activities included in the path {1, 2, 4, 8}}

Therefore S3 = {2, 5, 7}.

Path Sets

{(2, 3), (2, 5), (2, 6), (2, 7), (4, 3), (4, 5), (4, 6), (4, 7)}

Step 5: The possible 2-tuplets are: (2, 3), (2, 5), (2, 6), (4, 3), (4, 5), (4, 6), (4, 7).

Since each path set contains one element of each of the 2-tuplets (2, 3) and (4, 3) belong to, therefore the two 2-tuplets are added to the current solution:

S1 = {1, 8, 23, 34, 256, 257, 456, 457}.

The maximum number of activities in the cut sets generated q = 2 < total number of path sets n = 3. Create 3-tuplets from activities in S2 and S3.

Therefore S1 = {1, 8, 23, 34, 256, 257, 456, 457}.

Cut Sets

- 1, 8, 23, 34, 256, 257, 456, 457
- The reliability of a cut set is the product of the component reliabilities
- Example: R(Cut Set {1}) = R1
  R(Cut Set {2, 3}) = R2R3
Activity Number and Name
1 - Identify market needs
2 - Identify target cost and selling price of the product
3 - Develop a product
4 - Manufacturing system planning and design
5 - Develop a basic process plan for the product
6 - Develop alternative process plan for the product
7 - Modify the existing design of a manufacturing system
8 - Design a new manufacturing system
9 - Analyze the design of products and the system

The Reduction Approach
\[
R_{56} = R_5 + R_6 - R_5R_6 \\
R_{78} = P_7R_7 + P_8R_8 \\
R_s = R_1R_2R_3R_4R_5R_6R_7R_8R_9
\]

EXAMPLE

The Path Tree Algorithm

Reliability network of the IDEF3 model

Reliability tree

The Path Tree Algorithm

Reliability network of the IDEF3 model

Reliability tree

The path sets are:
\[
\{1, 2, 3, 4, 5, 7, 9\}, \{1, 2, 3, 4, 5, 8, 9\}, \{1, 2, 3, 4, 6, 7, 9\}, \{1, 2, 3, 4, 6, 8, 9\}.
\]
The path sets of the IDEF3 model are:

{1, 2, 3, 5, 10, 4, 7, 9}, {1, 2, 3, 5, 10, 4, 8, 9}, {1, 2, 3, 6, 10, 4, 7, 9},
{1, 2, 3, 6, 10, 4, 8, 9}, {1, 2, 3, 5, 10, 11, 4, 7, 9},
{1, 2, 3, 5, 10, 11, 4, 8, 9}, {1, 2, 3, 5, 10, 11, 6, 4, 7, 9},
{1, 2, 3, 5, 10, 11, 6, 4, 8, 9}, {1, 2, 3, 6, 10, 11, 4, 7, 9},
{1, 2, 3, 6, 10, 11, 5, 4, 7, 9}, {1, 2, 3, 6, 10, 11, 5, 4, 8, 9}
The path set - activity incidence matrix allow one not only to check the functional state of the model, but also to calculate the reliability of IDEF3 model under the current state of activities.

**Algorithm 4**

Step 1: Select the columns associated with activities that have failed in the model and draw a vertical lines \( v_j \) through each of these columns.

Step 2: For each \( R_j \) crossed by the vertical line \( v_j \) draw a horizontal line \( h_i \).

Step 3: Transform the incidence matrix \([P_{ij}]\) into \([P'_{ij}]\) by removing rows and columns corresponding to all the vertical and horizontal lines drawn in Step 1 and 2.

Step 4: If \([P'_{ij}]\) is empty, then stop. The model does not function. Otherwise, calculate \( r_i = \prod_{j} R_j \) for all \( i \) of matrix \([P'_{ij}]\).

Step 5: Calculate the reliability of the new IDEF3 model as \( R_i = 1 - (1 - r_i) \), where \( i \) ranges over all path sets in matrix \([P'_{ij}]\).

**Steps of Algorithm 4**

Step 1: Columns 2, 4, and 5 of matrix \([P_{ij}]\) are selected and vertical lines \( v_2, v_4, v_5 \) are drawn.

Step 2: Two horizontal lines \( h_1, h_2 \) are drawn. The results of Step 1 and 2 are presented in matrix (9).

Step 3: Matrix (9) is transformed into matrix (10).

Step 4: Since path set matrix (10) is not affected by the activity removal, therefore

\[ r_i = \prod_{j} R_j = R_1 \cdot R_2 \cdot R_4 \cdot R_8 \]

Step 5: The reliability of the new IDEF3 model is

\[ R_i = 1 - (1 - r_i) = R_1 \cdot R_2 \cdot R_4 \cdot R_8 \]
SUMMARY

• Four algorithms for reliability evaluation of IDEF3 models were discussed.

• Reliability analysis identifies critical activities in the process, improves the process performance, decreases downtime and operating cost of the process.