Qualitative Reasoning in Engineering Design

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Outline
• INTRODUCTION
• DEPENDENCY ANALYSIS IN CONSTRAINED DESIGN
  Problem Definition
  Dependency Network
  Design Examples
• QUANTITATIVE REASONING
• SUMMARY

Problem Definition
Types of constraints
Equations: Equations are used to represent engineering constraints, for example, \( F = ma \).
Inequalities: May represent engineering constraints corresponding to, e.g., design specifications or performance measures.
Qualitative constraints: A qualitative constraint represents relations of qualitative nature.
Computer-based procedures: These constraints are implicit and are usually in the form of a computer program that takes certain inputs and produce outputs.
Influence rules: Conditional statements specifying a constraint under certain conditions.

Design Problem Components
Design goals (objectives): A design goal is expressed with design variables and it captures the idea of “best” design.

Decision variables: A designer is allowed to make independent decisions on those variables.

Intermediate variables: The value of an intermediate variable is not directly determined by the designer, but by propagating the values of decision variables and design specifications through a set of constraints.

Design Problem-Solving Process
Specifications

Initial design

Manufacture
Reliability
Testing

Redesign

Analytic Evaluation

Acceptability

No

Yes

Done
**Dependency Network**

**Definition**

Dependency network is a four-tuple \( G = (V, E, \Omega, \Psi) \)

where:
- \( V \) is a set of vertices representing inequality constraints and design variables that are classified into three types:
  - \( V_D \): set of decision variables
  - \( V_I \): set of intermediate variables
  - \( V_P \): set of design goals
- \( E \) is a set of directed edges, a subset of \( V \times V \)
- \( \Omega \) is the set of qualitative dependencies on arcs
- \( \Psi \) is the set of quantitative dependencies on arcs

**Qualitative Dependency**

**Definition**

\[
\delta_{ab} = \begin{cases} 
+ & \text{if } \partial a = + \text{ and } \partial b = + \\
- & \text{if } \partial a = + \text{ and } \partial b = - \\
0 & \text{if } \partial a \in \{+, -\}, 0, ? \text{ and } \partial b = 0, \text{ when } \partial a \in \{+, -\}, 0, ? \text{ or } \partial b \notin \Omega
\end{cases}
\]

For \( \delta_{ab} \in \Omega \), where \( a, b \notin V \) and \( (a, b) \notin E \)

- \( \delta_{x} = + \) means that variable \( x \) is increasing
- \( \delta_{x} = - \) means that variable \( x \) is decreasing
- \( \delta_{x} = 0 \) means that variable \( x \) remains unchanged

**Parallel Inference**

**Definition**

Let \( G = (V, E, \Omega, \Psi) \) be a dependency network. Variables \( a, b, c \in V \) and \( \delta_{ac}, \delta_{bc} \in \Omega \), where \( a \) and \( b \) are directly incident to variable \( c \), and \( a \) and \( b \) are not directly linked.

The result of parallel inference \( \delta_{ac} \oplus \delta_{bc} \) is as follows:

\[
\begin{array}{cccc}
\delta_{ac} & \oplus & \delta_{bc} & \delta_{ac} \oplus \delta_{bc} \\
+ & - & 0 & ? \\
? & + & 0 & ? \\
0 & 0 & 0 & 0 \\
? & ? & ? & ?
\end{array}
\]

**Example: Parallel Inference**

\[
\begin{align*}
3a + b &= c \\
3a + 5 &= 14 & (+, + = +) \\
a \land b \lor 3a + 6 &= 12 & (-, + = +) \text{ OR} \\
3(-3) + 6 &= -3 & (-, + = -) \text{ ?}
\end{align*}
\]

**Serial Inference**

**Definition**

Let \( G = (V, E, \Omega, \Psi) \) be a dependency network. Variables \( a, b, c \in V \) and \( \delta_{ab}, \delta_{bc} \in \Omega \), where variable \( a \) is incident to variable \( b \), and \( b \) is incident to variable \( c \).

The result of serial inference \( \delta_{ab} \otimes \delta_{bc} \) is as follows:

\[
\begin{array}{cccc}
\delta_{ab} & \otimes & \delta_{bc} & \delta_{ab} \otimes \delta_{bc} \\
+ & - & 0 & ? \\
0 & 0 & 0 & 0 \\
? & ? & ? & ?
\end{array}
\]

**Example: Serial Inference**

\[
\begin{align*}
a \delta_{ab} & \rightarrow b \delta_{bc} \\
F &= 7a \\
P &= 2.5F
\end{align*}
\]

\[
\begin{align*}
7a &= F \\
2.5F &= P
\end{align*}
\]

from the formulas

\[
\begin{align*}
(a, F) + (F, P) + P & \text{ from the table}
\end{align*}
\]
What is the main benefit of qualitative reasoning?

Generally valid and easy to evaluate relations
Proposition 1

Let \( a \) and \( b \) be two design variables. The relationship between \( a \) and \( b \) is represented as "\( b \propto a^n \)" i.e., \( b \) is approximately proportional to \( a \) to the power of \( n \). The rate of change between \( a \) and \( b \) is expressed as

\[
\frac{\Delta b}{b} = n \frac{\Delta a}{a}
\]

Quantitative Dependency

Definition

Let \( G = (V, E, \Omega, \Psi) \) be a dependency network and variables \( a, b \in V(G) \) and the directed edge \((a, b) \in E(G)\). Quantitative dependency \( \psi_{ab} \) is defined as:

\[
\psi_{ab} = n
\]

where

\[
n = \frac{(\Delta b / b)(a)}{(\Delta a / a)(b)}
\]

Example

\( F = ma \); Nominal values \( F = 20 \), \( a = 5 \)

\[
\psi_{aF} = \frac{\Delta F / F}{\Delta a / a} = \frac{10\% F / F}{10\% a / a} = \frac{2/20}{.5/5} = 1
\]

Example 1: Car design

Constraints related to the performance design perspective

\[
a = \frac{F g}{(W_b + W_m)} \quad F = \frac{2 T r_s}{D} \quad T \propto W_m
\]

- \( a \): acceleration
- \( F \): the force to generate the acceleration required
- \( g \): the gravity acceleration constant
- \( W_b \): the weight of body of the car
- \( W_m \): engine weight
- \( T \): the torque of the engine
- \( r_s \): the gear ratio between the driving shaft and the wheel axle
- \( D \): the wheel diameter.
Constraints imposed by other perspectives

\[ g_1 = W_m - W_{m,\text{Lim}} \leq 0 \]
\[ g_2 = D_{\text{Lim}} - D \leq 0 \]
\[ g_3 = W_{b,\text{Lim}} - W_b \leq 0 \]

- \( W_m \) weight of the engine
- \( W_{m,\text{Lim}} \) upper limit on the engine weight
- \( D \) wheel diameter
- \( D_{\text{Lim}} \) lower wheel diameter limit
- \( W_b \) weight of the car body
- \( W_{b,\text{Lim}} \) lower weight limit of the car body

Dependency network for the car design problem

\[ a = \frac{F g}{(W_b + W_m)} \]
\[ F = \frac{2 T r_s}{D} \]
\[ T \propto W_m \]

In order to increase the acceleration of the car, the following strategies can be used:

1. Increase the value of gear ratio \((r_s)\).
2. Decrease the value of wheel diameter \((D)\). One can set the wheel diameter \((D)\) to its lower limit \((D_{\text{Lim}})\), as variable \(D\) does not influence other constraints.
3. Decrease the value of body weight \((W_b)\) of the car to its lower limit \((W_{b,\text{Lim}})\).

The qualitative dependency between engine weight \(W_m\) and acceleration \(a\) is unknown (?)

\[ \delta_{W_m,a} = (\delta_{F,g} \otimes \delta_{T,F} \otimes \delta_{W_m,T}) \otimes \delta_{Y,a} = ? \]

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