### Solution

In this problem, time rather than distance should be used as it is the primary object of optimization.

#### Iteration 1

**Step 0.** From the flow matrix \( f_{ij} \),

\[ \max \{ f_{ij} : i = 1, 2, 3, 4, 5; j = 1, 2, 3, 4, 5 \} = f_{23} = 72 \]

is obtained. Thus \( i^* = 2, j^* = 3 \) is determined.

Machines 2 and 3 are connected and included in the solution.

**Consider two alternatives:**

(a) \[ \begin{array}{ccc} 1 & 2 & 3 \end{array} \]

(b) \[ \begin{array}{ccc} 2 & 3 & 1 \end{array} \]

**Compute:**

\[ q_1 s^* t_1 = q_{12} t_{12} t_{13} t_{12} + q_{13} t_{13} t_{12} t_{13} + q_{14} t_{14} t_{13} t_{12} + q_{15} t_{15} t_{14} t_{13} t_{12} = 50(9) + 25(9+2) = 50(25) + 50(25) = 725 \]

Since \( q_{31} < q_{12} \), alternative (b) is selected; machine 1 is placed to the right of machine 3.
Consider two alternatives:

\[ q_j^s = q_5^2 = f_5^2(3) + f_5^3(5+2) + f_5^1(5+2+2) = 702 \]

\[ q_j^s = q_5^1 = f_5^1(4) + f_5^3(4+2) + f_5^2(4+2+2) = 804 \]

As \( q_5^1 > q_5^2 \), alternative (a) is selected; machine 5 is placed left of machine 2.

Iteration 3
Step 1. Compute \( \max \{ f_{14}, f_{54} \} = f_{14} = 45 \)
Set \( s^* = 4 \).
Consider two alternatives:

(a) \[
\begin{array}{cccc}
4 & 5 & 2 & 3 \\
1 & & & 1
\end{array}
\]

(b) \[
\begin{array}{cccc}
5 & 2 & 3 & 1 \\
4 & & & 4
\end{array}
\]

Compute:

\[ q_j^s = q_{45} = f_{45}^1 + f_{45}^3(f_{45} + f_{52}) + f_{45}^4(f_{45} + f_{52} + f_{23}) + f_{45}^1(f_{45} + f_{52} + f_{23} + f_{31}) \]
\[ = 4(5) + 2(5+3) + 14(5+3+2) + 21(5+3+2+3) = 821 \]

Since \( q_{14} < q_{45} \), alternative (b) is selected; machine 4 is placed to the right of machine 1.

The Layout of Machines for Problem 1

Determine the double-row layout of machines, given the data below for five machines:

(a) Matrix of frequency of trips
\[ [c_{ij}] = \begin{bmatrix}
1 & 0 & 0 & 3 & 0 \\
2 & 30 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 \\
7 & 0 & 0 & 0 & 0
\end{bmatrix} \]

(b) Machine dimensions

<table>
<thead>
<tr>
<th>Machine Number</th>
<th>Dimension x, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6 x 2</td>
</tr>
<tr>
<td>2</td>
<td>3 x 2</td>
</tr>
<tr>
<td>3</td>
<td>3 x 2</td>
</tr>
<tr>
<td>4</td>
<td>7 x 3</td>
</tr>
<tr>
<td>5</td>
<td>4 x 2</td>
</tr>
</tbody>
</table>

(c) Clearance matrix
\[ [c_{ij}] = \begin{bmatrix}
1 & 2 & 1 & 2 & 3 \\
2 & 3 & 0 & 0 & 1 \\
3 & 4 & 0 & 0 & 0 \\
5 & 4 & 0 & 0 & 0
\end{bmatrix} \]
Assumptions:
• The travel time between any pair of machines is proportional to the corresponding rectilinear distance
• The isle width is 2 units
• Machines are arranged lengthwise along the isle

Note: li denotes the length of machine i and wi denotes the width of machine i.

Distance measures
Consider 3 machines

Unidirectional distance (along x axis)
\( u_{13} = 0, \quad u_{12} = 7, \quad u_{23} = u_{12} = 7 \)

Rectilinear distance
\( d_{13} = 3, \quad d_{12} = 7, \quad d_{23} = 7 + 3 = 10 \)

Solution
The matrix of unidirectional distances is calculated using the machine dimensions and clearances.

(b) Machine dimensions

<table>
<thead>
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<td>7 x 3</td>
</tr>
<tr>
<td>5</td>
<td>4 x 2</td>
</tr>
</tbody>
</table>

(c) Clearance matrix

\[
[c_{ij}] = \begin{bmatrix}
0 & 3 & 1 & 2 & 1 \\
0 & 4 & 1 & 2 \\
0 & 3 & 1 \\
0 & 2 \\
0 & \\
\end{bmatrix}
\]

The rectilinear distance between machines 2 and 3 is equal to the isle width, \( d_{23} = 2 \).

Assume the travel time \( t_{23} = d_{23} = 2 \).

Iteration 1
Step 0. \( \max \{ f_{ij} : i = 1, 2, ..., 5, j = 1, 2, ..., 5 \} = f_{23} = 80 \) is determined.
Thus \( i^* = 2 \) and \( j^* = 3 \).

Machines 2 and 3 are assigned to the opposite sites of the isle and included in the solution.

\[
[f_{ij}] = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
0 & 50 & 40 & 20 & 60 \\
0 & 80 & 10 & 20 & 40 \\
0 & 15 & 60 & 0 & 75 \\
0 & 15 & & & \\
\end{bmatrix}
\]

Step 1. Compute \( \max \{ f_{2k}, f_{3l}, f_{5v} : k, l, v = 1, 4 \} = f_{51} = 60 \).
Thus \( t^* = 1 \) and \( j^* = 1 \).

Machines 2 and 3 are assigned to the opposite sites of the isle and included in the solution.

\[
[f_{ij}] = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
0 & 50 & 40 & 20 & 60 \\
0 & 80 & 10 & 20 & 40 \\
0 & 15 & 60 & 0 & 75 \\
0 & 15 & & & \\
\end{bmatrix}
\]

Set \( t^* = 1 \).
Consider two alternatives:

\( (a) \quad (b) \)

\[
\begin{align*}
\text{(a)} &: 4 & 2 & 5 \\
\text{(b)} &: 2 & 5 & 1
\end{align*}
\]

Since \( q_{2531} < q_{2135} \), alternative (a) has to be considered in the next step.

Set \( U = \{1, 2, 3, 5\} \)

Step 2. Set \( c^* = 4 \)

Consider four alternatives for this solution from step 1.

\( (c) \quad (d) \)

\[
\begin{align*}
\text{(c)} &: 2 & 5 & 4 \\
\text{(d)} &: 2 & 5 & 1
\end{align*}
\]

Based on solution \( q_{2531} \)

\[
\begin{align*}
\text{(a)} &: 4 & 2 & 5 \\
\text{(b)} &: 2 & 5 & 1 \\
\text{(c)} &: 2 & 5 & 4 \\
\text{(d)} &: 2 & 5 & 1
\end{align*}
\]

Since \( q_{42} = \min \{q_{42}, q_{43}, q_{54}, q_{14}\} \), alternative (a) is selected.