Problem 1 (10%)
Transform number $x$ from the interval $[5, 8]$ into number $y$ from the interval $[10, 17]$.

Problem 2 (90%)
Figure 1 shows the relationship between customer requirements and subsystems of an electrical product in the form of a house of quality. The customer evaluations as well as the current (product model A) and desired (product model B) technical evaluations are provided in the house of quality. The ranges for each requirement and each technical subsystem are identical integer numbers in the interval $[1, 5]$. The numerical scale for the relationships in the main QFD matrix is (strong positive = 9, medium positive = 3, weak positive = 1).

1. Determine the values of customer requirements that would meet the new levels of technical product evaluations.
2. For the following values of customer requirements [easy to read = 5, long life = 4, easy to adjust = 4, and easy to carry = 4], determine the level of technical product evaluations.
Solution:

1. Transform number $x$ from the interval $[5, 8]$ into number $y$ from the interval $[10, 17]$.

In order to transform the number $x$ from the scale $[5,8]$ into the scale $[10,17]$ we need to perform two transformations.

**Transformation Equation**

If:

- $x[a,b]$
- $x'[c,d]$
- $x''[e,f]$

Then,

$$X' = c + \frac{(x-a)}{(b-a)} \cdot (d-c)$$

And,

$$X'' = e + (f-e) \cdot X'$$

**Transformation 1:**

$X[5,8]$ into $X'[0,1]$

$$X' = \frac{(x-a)}{(b-a)}$$

$$X' = \frac{(x-5)}{(8-5)}$$

$$x' = \frac{(x-5)}{3}$$

**Transformation 2:**

$X'[0,1]$ into $X''[10,17]$
Problem 2

1. Determine the values of customer requirements that would meet the new levels of technical product evaluations.

General Procedures to solving this problem
a. Transform Xs from [1,5] into X’s [0,1]
b. Transform X’s int Y’s (both in the [0,1] scale)
c. Transform Y’s [0,1] into Ys [1,5]
So to the specifics:

a. X [1, 5] domain transformed to X' [0, 1] domain

New technical evaluations
X in [1, 5] domain       X' in [0, 1] domain
X1=5         X1' = (5-1)/(5-1) =1
X2=5    X2' = (5-1)/(5-1) =1
X3=4     X3' = (4-1)/(5-1) =.75

b. X' domain transformed to Y' domain (Then Solve)

Y'1= (9X'1 + 3 X'3+ X'1* X'3) / (13-0) = 0.9231
Y'2= (3 X'2+ 3 X'3+ X'2 X'3) / (7-0) = 0.8571
Y'3= (1 X'3+ 9X'2–X'1 X'3) / (9-0) = 1
Y'4= (3 X'2 + 9X'1 + X'2 X'3) / (13-0) = 0.8077
The denominators are the range of the Y’s. They are calculated by determining the min and max of the Y’ given the possible X’ values, in this case [0,1].

e.g., Y1’s max is given when the X’s are equal to 1; 13. The min is given when X’s are 0; 0.

c. Transform Y’s [0,1] into Ys [1,5]

Y1 = 1 + 0.9231(5-1) = 4.6924
Y2 = 1 + 0.857(5-1) = 4.43
Y3 = 1 + 1(5-1) = 5
Y4 = 1 + 0.807(5-1) = 4.23

Final Solution:
Given new levels of requirements, the corresponding levels of technical evaluations are:
X1 = 5  Y1 (easy to read) = 4.69  
X2 = 5  Y2 (long life) = 4.43  
X3 = 4  Y3 (easy to adjust) = 5  
Y4 (easy to carry) = 4.23

2. For the following values of customer requirements [easy to read = 5, long life = 4, easy to adjust = 4, and easy to carry = 4], determine the level of technical product evaluations.

General Procedure for solving this problem
We are essentially reversing the process from part 1 (kind of). We know our customer requirements Y and are trying to calculate our product evaluations X.

a. From Ys calculate equations for X’s
b. Solve the system of nonlinear equations
c. Transform X’s [0,1] into Xs [1,5]

A.
For this calculation we use the equation from Case Study 5 slide 9
Y1= 1+(5-1)*Y1  = 5

Then plugging in the Y’s into the equations is shown as:
Y1= 1+ (5-1)* (9X’1 + 3 X’3+ X’1* X’3) / 13 = 5
Y2= 1+ (5-1)* (3 X’2+ 3 X’3+ X’2 X’3) / 7 = 4
Y3= 1+ (5-1)* (1 X’1+ 9X’2 –X’1 X’2) / 9 = 4
Y4= 1+ (5-1)* (3 X’2 + 9X’3 + X’2 X’3) / 13 = 4

By rearranging terms we get:
(9X’1 + 3 X’3+ X’1* X’3) = 13
(3 X’2+ 3 X’3+ X’2 X’3) = 5.25
(1 X’1+ 9X’2 –X’1 X’2) = 6.75
(3 X’2 + 9X’3 + X’2 X’3) = 9.75
We have a system of four nonlinear equations with three unknown variables \( x \). An optimization problem should be formulated and solved with LINGO. We need to first get the equations into the format that LINGO requires:

So our equations become

\[
9x_1 + 3x_3 + x_1 \times x_3 \geq 13 \\
\text{Similarly for the } 2^{\text{nd}} - 4^{\text{th}}.
\]

Then Input then becomes:

**LINGO INPUT:**

```
max= x1+x2+x3;
9 * x1 + 3 * x3 + x1 * x3 <= 13;
3* x2 + 3 *x3 + x2 * x3 <= 5.25;
1 * x1 + 9 * x2 - x1 * x2 <= 6.75;
3 * x2 + 9 * x3 + x2 * x3 <= 9.75;
x1 >= 0;
x2 >= 0;
x3 >= 0;
x1 <= 1;
x2 <= 1;
x3 <= 1;
end
```

Local optimal solution found at iteration: 10
Objective value: 2.500100

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So,

\[
x_1' = 1 \\
x_2' = .7187500 \\
x_3' = .7813505
\]

We then need to scale those values back to the \([1,5]\) scale

\[
x_1 = 1 + (5-1) * 1 = 5 \\
x_2 = 1 + (5-1) * .7187500 = 3.875
\]
X3 = 1 + (5-1)*.7813505 = 4.1254

So the technical product evaluations that give us the customer requirements [5, 4, 4, 4] are [5, 3.875, 4.125].

**Verification**

We can see how far off from our customer requirements we are based on the computed technical requirements.

\[
(9*1 + 3 *.78135+ 1* .78135)= 12.1254
\]

\[
(3* .71875+ 3 *.78135+.71875 *.78135) =5.061895
\]

\[
(1 *1+ 9*.71875 –1*.71875) =6.46875
\]

\[
(3 *.71875 + 9*.78135 +.71875 *.78135) =9.75999
\]

So the actual customer requirements are:

\[
Y_1= 1+ (5-1)*(12.13) / 13 = 4.73 \sim 5
\]

\[
Y_2= 1+ (5-1)*5.06 / 7 = 3.89 \sim 4
\]

\[
Y_3= 1+ (5-1)* 6.47 / 9 = 3.87 \sim 4
\]

\[
Y_4= 1+ (5-1)*9.76 / 13 = 4.00 = 4
\]