Problem 1

A mechanical engineer has designed the part in Fig. A2. The dimensions A through H have been assigned values shown in the first column of Table A1. The tolerances for the three dimensions F, G, and H are imposed by the part of the main subassembly. (see Table A1)

Figure A2 Side view of an mechanical part

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Tolerances</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 10</td>
<td>tA = ±15</td>
<td>cA = 20</td>
</tr>
<tr>
<td>B = 10</td>
<td>tB = ±15</td>
<td>cB = 6</td>
</tr>
<tr>
<td>C = 12</td>
<td>tC = ±15</td>
<td>cC = 5</td>
</tr>
<tr>
<td>D = 14</td>
<td>tD = ±15</td>
<td>cD = 10</td>
</tr>
<tr>
<td>E = 18</td>
<td>tE = ±15</td>
<td>cE = 50</td>
</tr>
<tr>
<td>F = 30</td>
<td>tF = ±15</td>
<td>cF = 6.5</td>
</tr>
<tr>
<td>G = 30</td>
<td>tG = ±15</td>
<td>cG = 30</td>
</tr>
<tr>
<td>H = 70</td>
<td>tH = ±15</td>
<td>cH = 40</td>
</tr>
</tbody>
</table>

Table A1 Dimensions, Tolerance, and Costs

Solutions:

Given the candidate tolerances for the dimensions A through E and the corresponding costs in Table A1, determine with LINDO the optimal tolerances for dimensions A through E.

What would you recommend if you had to reduce the total tolerance cost below the value obtained in (a)?
Solve the model with LINDO,

OBJECTIVE FUNCTION VALUE

1)  99.00000

VARIABLE          VALUE          REDUCED COST
X11         0.000000         20.000000
X12         1.000000         14.000000
X21        1.000000         25.000000
X22         0.000000         19.000000
X31         1.000000         10.000000
X32         0.000000         5.500000
X41         1.000000         10.000000
X42         0.000000         10.000000
X43         0.000000         6.500000
X51         1.000000         0.000000

The solution obtained from LINDO is:

\[ X_{12} = X_{21} = X_{31} = X_{41} = X_{51} = 1 \]

which means we set tolerance of dimension A to t_{12} (±3 units), dimension B to t_{21} (±5 units), dimension C to t_{31} (±2 units), and dimension D to t_{41} (±2 units).

- Higher accuracy generally requires higher manufacturing cost. To reduce the cost, we can set a higher tolerance for a dimension chain, so that larger tolerances of dimensions in the chain can be used. For example, set the tolerance of F to a greater number 9, we may then use t_{22}, which will reduce the total cost by 6.

Problem 2

Three products P1, P2, and P3 were evaluated based on three criteria and a number of subcriteria. The average scores assigned by the teams to each product and each subcriterion are listed in Table A1. To evaluate the weights assigned to the criterion and subcriterion, a team of four experts provided preferences R1 through R16 based on the preference function:

\[
\mu_{\text{most}}(X) = \begin{cases} 
1 & \text{for } x \geq 0.8, \\
2x - 0.6 & \text{for } 0.3 < x < 0.8, \\
0 & \text{for } x \leq 0.3.
\end{cases}
\]

Preferences for criteria A, B, and C are:

\[
R_1 = \begin{pmatrix} 
-0.6 & 0.1 \\
0.9 & 0.7 & -
\end{pmatrix} \\
R_2 = \begin{pmatrix} 
0.5 & 0.3 & 0.7 \\
0.7 & 0.5 & -
\end{pmatrix} \\
R_3 = \begin{pmatrix} 
-0.4 & 0.8 \\
0.2 & 0.8 & -
\end{pmatrix} \\
R_4 = \begin{pmatrix} 
-0.5 & 0.1 \\
0.9 & 0.4 & -
\end{pmatrix}
\]

Preferences for subcriteria a1 through a4 are

\[
R_5 = \begin{pmatrix} 
-0.6 & 0.1 \\
0.9 & 0.7 & -
\end{pmatrix} \\
R_6 = \begin{pmatrix} 
0.3 & 0.7 & 0.5 \\
0.7 & 0.5 & -
\end{pmatrix} \\
R_7 = \begin{pmatrix} 
0.4 & 0.8 \\
0.2 & 0.8 & -
\end{pmatrix} \\
R_8 = \begin{pmatrix} 
-0.5 & 0.1 \\
0.9 & 0.4 & -
\end{pmatrix}
\]

Table A1. List of Subcriteria and the corresponding Numerical Scores of Products

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Number of team members</th>
<th>Subcriterion</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. User-friendliness</td>
<td>4</td>
<td>A1: Easy to open</td>
<td>80</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A2: Easy to close</td>
<td>100</td>
<td>80</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A3: Stays open</td>
<td>100</td>
<td>80</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A4: Stays closed</td>
<td>55</td>
<td>35</td>
<td>55</td>
</tr>
<tr>
<td>B. Effectiveness</td>
<td>4</td>
<td>B1: Wide opening</td>
<td>10</td>
<td>90</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B2: No obstructions</td>
<td>100</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B3: Glass visibility</td>
<td>90</td>
<td>70</td>
<td>10</td>
</tr>
<tr>
<td>C. Safety</td>
<td>4</td>
<td>C1: Easy to turn</td>
<td>100</td>
<td>80</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C2: Easy to stop</td>
<td>90</td>
<td>100</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C3: Frostproof</td>
<td>55</td>
<td>35</td>
<td>50</td>
</tr>
</tbody>
</table>
Preferences for subcriteria b1 through b3 are

$R_{b1} = \begin{pmatrix} -0.6 & 0.7 & 0.3 \\ 0.4 & 0.3 & 0.9 \\ 0.7 & - & - \end{pmatrix}$

$R_{b2} = \begin{pmatrix} 0.5 & - & - \\ 0.3 & 0.7 & - \\ 0.5 & 0.7 & - \end{pmatrix}$

$R_{b3} = \begin{pmatrix} -0.6 & 0.8 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.9 & 0.4 & - \end{pmatrix}$

Preferences for subcriteria c1 through c3 are

$R_{c1} = \begin{pmatrix} -0.6 & 0.1 & 0.4 \\ 0.4 & 0.3 & 0.9 \\ 0.9 & 0.7 & - \end{pmatrix}$

$R_{c2} = \begin{pmatrix} -0.7 & 0.3 & 0.3 \\ 0.3 & 0.5 & 0.7 \\ 0.7 & 0.5 & - \end{pmatrix}$

$R_{c3} = \begin{pmatrix} -0.4 & 0.8 & 0.6 \\ 0.6 & 0.2 & 0.2 \\ 0.2 & 0.8 & 0.9 \end{pmatrix}$

• Select a product with the maximum original score
• Select a product with the maximum revised score

Solution

• Without considering the preferences of the experts, product 1 will be the best choice based on the original scores.

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>375</td>
<td>230</td>
<td>250</td>
</tr>
<tr>
<td>B</td>
<td>250</td>
<td>200</td>
<td>155</td>
</tr>
<tr>
<td>C</td>
<td>275</td>
<td>240</td>
<td>180</td>
</tr>
<tr>
<td>Total</td>
<td>900</td>
<td>680</td>
<td>620</td>
</tr>
</tbody>
</table>

• Find the weight vector for sub criterion A:

$R_a = \begin{pmatrix} -1/2 & 1/2 & 1/3 \\ 1/2 & -1/2 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$

And hence, $[g_a] = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/3 \end{pmatrix}$

$Z_{a\text{most}} = [0.4, 0.4, 0.4]$.

Then, we obtain the weights: $[w_a] = [0.33, 0.33, 0.33]$.
Find the weight vector for sub criterion B:

\[ \mathbf{R}_b = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \]

And hence, \[ [g_{bij}] = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \]

\[ [g_{bi}] = [1/2, 1, 0] \]

\[ Z_{b_{\text{most}}} = [0.4, 1, 0] \]

Then, we obtain the weights: \[ [w_{bi}] = [0.268, 0.714, 0] \]

Find the weight vector for sub criterion C:

\[ \mathbf{R}_c = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \]

And hence, \[ [g_{cij}] = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \]

\[ [g_{ci}] = [1/2, 1/2, 1/2] \]

\[ Z_{c_{\text{most}}} = [0.4, 0.4, 0.4] \]

Then, we obtain the weights: \[ [w_{ci}] = [0.33, 0.33, 0.33] \]

The revised table of scores is as follows:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Weight</th>
<th>Subcriteria</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. User-friendliness</td>
<td>0.455</td>
<td>A1: Easy to open</td>
<td>20.64</td>
<td>25.68</td>
<td>49.8</td>
</tr>
<tr>
<td></td>
<td>0.454</td>
<td>A2: Easy to close</td>
<td>45.3</td>
<td>27.3</td>
<td>27.3</td>
</tr>
<tr>
<td></td>
<td>0.082</td>
<td>A3: Stays open</td>
<td>4.2</td>
<td>1.8</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>0.082</td>
<td>A4: Stays closed</td>
<td>3.9</td>
<td>2.1</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>Weighted subtotal</td>
<td>50.82</td>
<td>67.9</td>
<td>79.5</td>
</tr>
<tr>
<td>B. Effectiveness</td>
<td>0.286</td>
<td>B1: Wide opening</td>
<td>17.16</td>
<td>14.3</td>
<td>18.5</td>
</tr>
<tr>
<td></td>
<td>0.714</td>
<td>B2: No obstacles</td>
<td>71.4</td>
<td>57.1</td>
<td>57.1</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>B3: Good reality</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>Weighted subtotal</td>
<td>88.56</td>
<td>71.4</td>
<td>75.1</td>
</tr>
<tr>
<td>C. Safety</td>
<td>0.31</td>
<td>C1: Easy to turn</td>
<td>33</td>
<td>19.8</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>0.31</td>
<td>C2: Easy to step</td>
<td>29.7</td>
<td>33</td>
<td>28.4</td>
</tr>
<tr>
<td></td>
<td>0.31</td>
<td>C3: Freezeproof</td>
<td>28.05</td>
<td>29.7</td>
<td>29.7</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>Weighted subtotal</td>
<td>90.75</td>
<td>82.5</td>
<td>66.4</td>
</tr>
<tr>
<td>Weighted Total</td>
<td></td>
<td></td>
<td>107.976</td>
<td>88.728</td>
<td>85.456</td>
</tr>
</tbody>
</table>

- The weighted total scores can be obtained by multiply the subtotals on the table by the weight vector in step 1.
- The revised scores for the three products are: [107.976, 88.728, 85.456]

Therefore, product P1 will still be the best choice.