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RIVER MEANDERS

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ABSTRACT

Most river curves have nearly the same value of the ratio of curvature radius to channel width, in the range of 2 to 3. Meanders formed by meltwater on the surface of glaciers, and by the main current of the Gulf Stream, have a relation of meander length to channel width similar to rivers. Because such meanders carry no sediment, the shapes of curves in rivers are evidently determined primarily by the dynamics of flow rather than by relation to debris load.

Velocity distributions along river curves provide a generalized picture of flow characteristics. Evidence on flow resistance in curved channels suggests that a basic aspect of meander mechanics may be related to the distribution of energy loss provided by a particular configuration or curvature. No general theory of meanders is as yet satisfactory, however; in fact, present evidence suggests that no single theory will explain the formation and characteristics of all meanders and that few of the physical principles involved have yet been clearly identified.

RÉSUMÉ

Dans la plupart des courbes de rivières la valeur du rapport entre le rayon de courbure et la largeur du lit, de l'ordre de grandeur de 2 à 3, est presque toujours la même. Les méandres formés par l'eau de fonte à la surface des glaciers et par le principal courant du Gulf Stream, montrent une relation entre la longueur du méandre et la largeur du chenal semblable á celle des rivières. Puisque de tels méandres ne transportent pas de sédiments, on peut en conclure que la forme des courbes de rivières est determinée principalement par la dynamique de l'écoulement d'eau plutôt que liée à la charge sédimentaire.

La distribution des vélocités le long des courbes de rivières donne une image générale des caracteristiques d'écoulement. Ce qu'on sait de la résistance à l'écoulement dans les chenaux courbes suggère que la résistance minimum est un facteur de base dans la mécanique des méandres. Toutefois, aucune théorie générale des méandres n'est encore complètement satisfaisante. En fait, les données actuelles suggèrent qu'aucune théorie ne suffira pas à expliquer tous les méandres, et que quelques-uns seulement des principes de physique en jeu ont déjà été reconnus.

ZUSAMMENFASSUNG

Die meisten Flußwindungen haben etwa dasselbe Wertverhältnis von Krümmungsradius zu Bettbreite, ungefähr zwei zu drei. Mäanderwindungen, die durch Schmelzwasser auf der Oberfläche der Gletscher entstehen, und solche, die durch die Hauptströmung des Golfstromes bedingt sind, haben ein Verhältnis von Mäanderlange zu Bettbreite, welches dem der Flüsse ähnlich ist. Da solche Mäander kein Sediment mit sich führen, sind die Formen der Flußwindungen offensichtlich in erster Linie durch die Dynamik der Strömung bedingt, weniger durch die Gegebenheiten des mitgeführten Materials.

Die Verteilung der Strömungsgeschwindigkeit entlang den Flußwindungen zeigt ein verallgemeinertes Bild der Strömungsmerkmale. Was über den Strömungswiderstand in gewundenen Flußbetten beobachtet wurde, läßt die Vermutung aufkommen, daß der geringste Widerstand eine wichtige Ursache für Mäanderbildungen ist. Tatsachlich lassen die gegenwärtig beobachteten Tatsachen vermuten, daß vorläufig noch keine einzige Theorie alle Mäanderbildungen erklaren kann, und daß nur wenige der physikalischen Gesetzmäßigkeiten, die dabei eine Rolle spielen, bisher gefunden worden sind.

МЕАНДРЫ РЕК

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Резюме

Большинство речных излучин обладает почти одинаковой величиной отношения радиуса кривизны к ширине канала, колеблющейся в порядке от 2 до 3. Меандры, образованные талой водой на поверхности ледников и излучины главного течения Гольфштрема, обладают отношением длины излучины к ширине канала, подобным тому, которое наблюдается в реках. Ввиду того, что такие меандры не содержат осадков, формы излучин в реках, очевидно, определяются в первую очередь динамикой течения, нежели зависимостью от наличия кластического материала.

Распределение скорости течения вдоль речных излучин дает обобщенную характеристику течения. Доказательством сопротивления течения в криволинейных каналах наводит на мысль о том, что основным фактором в механике образования меандр является минимальное сопротивление. Еще не выработана общая удовлетворительная теория о происхождении меандр. В действительности, современные данные дают возможность сказать, что отдельная теория не даст объяснения происхождения всех типов меандр. Еще не изучены многие физические факторы, влияющие на происхождение меандр.

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General Statement

The most characteristic features of all stream channels, regardless of size, are the absence of long straight reaches and the presence of frequent sinuous reversals of curvature. These bends, whether or not they display regular reversals and sufficient symmetry to warrant the name meander, tend to be scaled versions of a given set of proportions. The proportions are determined by three dimensions: the repeating distance or length of the curve, the width of the channel, and the radius of curvature. In brief, the size of the bend appears to be proportional to the size of the river; large rivers have large bends, and small rivers have small bends.

An adequate theory of meanders should explain why all river bends seem to be of proportional sizes, and by what mechanical or physical principle a bend becomes adjusted in size to the stream discharge. Such a theory should also explain why some rivers meander and others do not. It should explain why meandering rivers occur on ice with high velocity of flow and no sediment load, and why tortuous channels are so prevalent in tidal estuaries. A meander theory should also explain how chance perturbations of the flow may persist and lead to the development of a sinuous pattern.

The present paper does not provide any theory. An attempt is made to analyze and integrate what appear to be the most prevalent essential characteristics of meandering channels in nature. At least some of these prevalent characteristics are probably basic to the mechanics of meandering channels which are as yet poorly understood. These include the external features of the channel, as well as the distribution of velocity and the pattern of flowing water. Such a summary may prove useful in the eventual definition of the necessary and sufficient conditions for meandering of channels in diverse natural environments.

There is a large published record on the occurrence and physiographic setting of meandering channels, as well as on the form and theory of river curves. It is not our purpose here to cite this record exhaustively, but rather to sift out those facts which appear to us most pertinent, emphasizing at the same time that opinions may differ as to what is pertinent.

Our studies of meandering streams and of meander mechanics lead us to write a somewhat different kind of a paper than might usually be expected in a "review" series. Many new observations are available, some previously unpublished, some recently published or in process of publication. We hope it will be of some service, especially to the reader who has an interest but not a specialty in the subject, to have these new observations summarized and related to earlier observations. We believe that the status of the problem will be clearer as a result, and these observations are, in our opinion, of some importance in indicating directions toward which future work might profitably be directed.

We then attempt to show that the problem of river meanders can be broken down into two parts. The first concerns the mechanical and hydraulic processes that govern the form, size, and probably the occurrence of meanders. The second includes the physiographic history of a particular channel and the formation, maintenance, and possible dissolution of a meandering pattern. Although these two parts are almost inseparable they do represent quite different ways of viewing the meander problem. We hope it will be clear from this review that many important physiographic questions cannot be answered with satisfaction because there is insufficient knowledge at present about meander mechanics.

The authors are much indebted to J. Hoover Mackin and W. W. Rubey for many constructive suggestions and for a critical reading of an early draft of the manuscript.

Geometry and Patterns of Channel Bends

Because most channels are not straight but sinuous, the forms which they display in plan view are, in detail, infinitely varied. In our experience it is unusual for a reach of natural stream to be straight for a distance exceeding 10 channel widths (Leopold and Wolman, 1957, p. 53). Yet certain common features are discernible. These features, common to many individual bends, probably have some significance in the mechanics or hydraulics of meanders, but this significance is only poorly understood. The geometric characteristics of a channel seen in plan view are defined by Figure 1.

We use the term "arc distance" to mean the length measured along the channel center line from one point of inflection to the next. "Sinuosity" we use to mean the ratio of arc distance to half the meander length.

The geometry of meander curves has been the object of extensive statistical study, examples of which are papers by Jefferson (1902), Inglis (1937; 1949, Pt. 1, p. 143), Bates (1939), and Leopold and Wolman (1957, p. 58). An additional independent sample in the appendix includes measurements of rivers in which the presence of a well-defined flood plain indicates that the channel form and pattern have developed reasonably free of bedrock control. These data for rivers of moderate to large size were derived from published topographic maps and for small streams from plane-table maps made by the authors. The appendix also includes some data from published laboratory studies of meandering streams in erodible materials.

Studies of the geometry of the patterns of meandering rivers give generally similar results.

The most consistent correlations are between meander length, channel width, and radius of curvature, as shown by Figure 2, plotted from the data in the appendix. Because of the variability in the dimensions of channels in nature, considerable scatter of plotted data is expected. Despite the scatter the relations among the factors appear to hold through a very large range of stream size, from laboratory streams a foot wide to the Mississippi River nearly a mile wide. Various authors have expressed the relations for rivers in alluvial flood plains by regression equations, a few of which are presented in Table 1. Table 1 also includes relations derived from the data in the appendix.

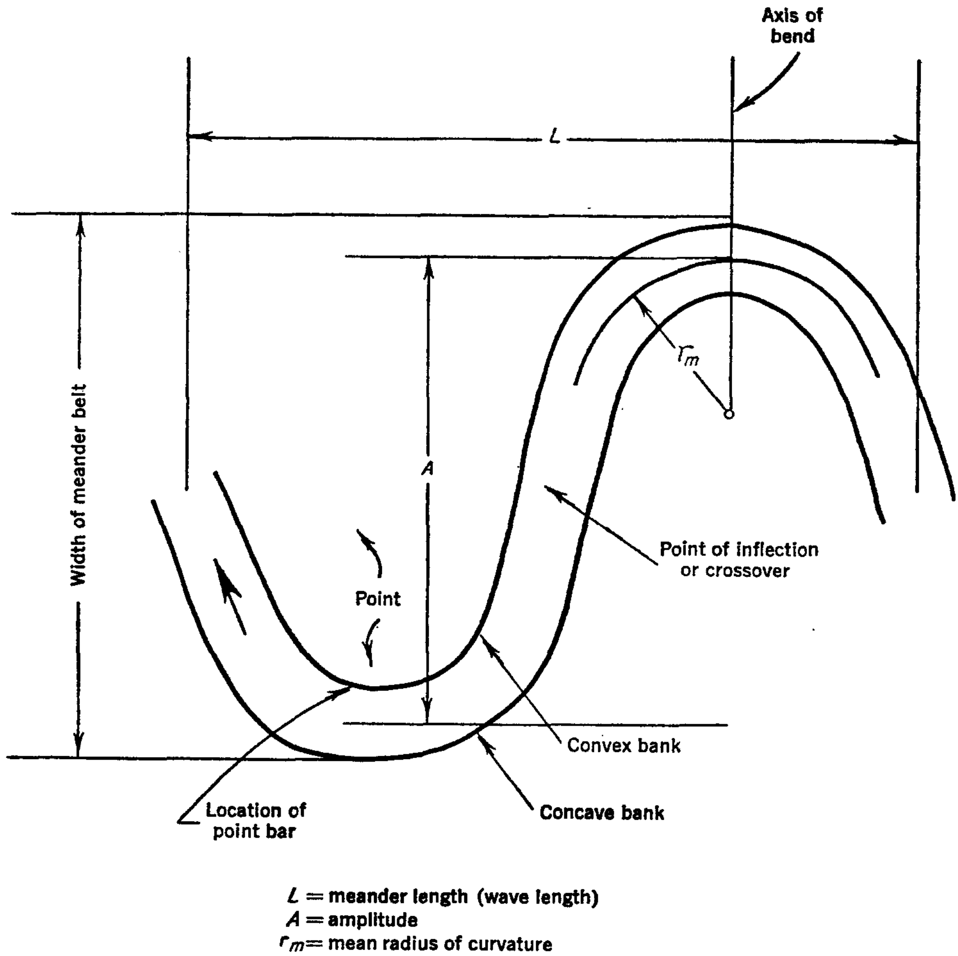

FIGURE 1.—SKETCH TO DEFINE TERMS USED IN DESCRIBING GEOMETRIC CHARACTERISTICS OF A MEANDERING CHANNEL

The exponents in the regression equations are all so close to unity that the relations between meander length (wave length), amplitude, radius of curvature, and channel width may be considered linear. Among different groups of data fairly consistent values are obtained for the coefficient relating meander length to channel width; meander length ranges from 7 to 10 times the channel width. This ratio holds in general not only for meanders on a broad flood plain but also characterizes sinusoidal or meandering valleys incised into consolidated materials (Dury, 1958, p. 109).

A marked inconsistency is exhibited, however, in the ratio of amplitude to channel width. Amplitude correlates only poorly with meander length. We tentatively interpret these findings to mean the amplitude of

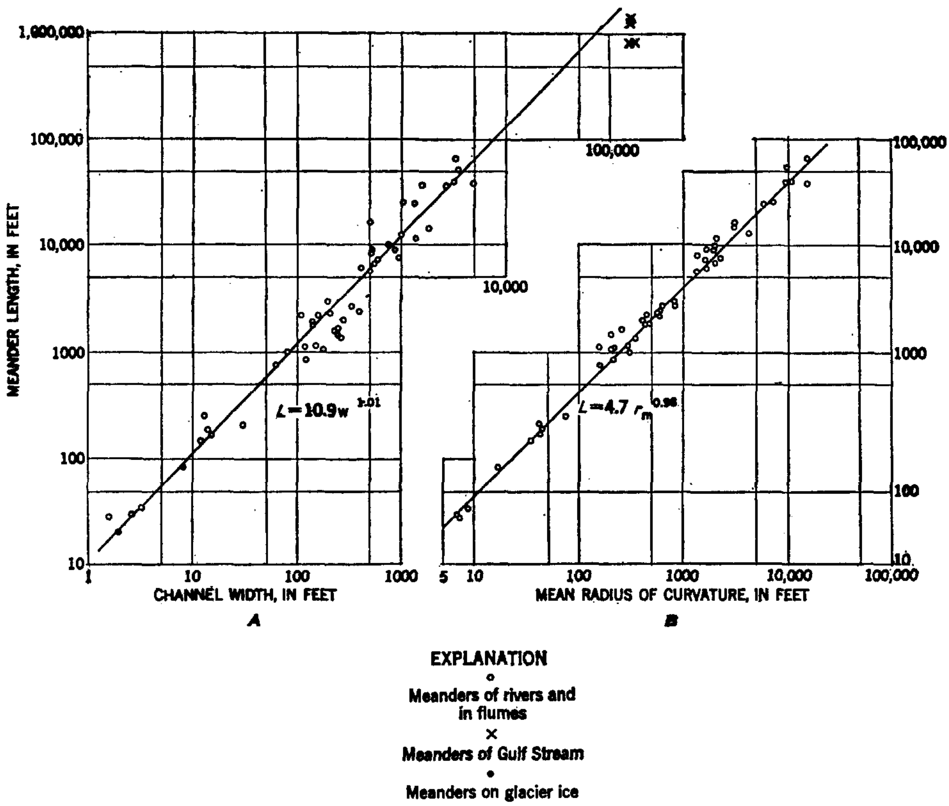

FIGURE 2.—RELATIONS BETWEEN MEANDER LENGTH AND CHANNEL WIDTH (LEFT DIAGRAM) AND MEAN RADIUS OF CURVATURE (RIGHT DIAGRAM)

TABLE 1.—EMPIRICAL RELATIONS BETWEEN SIZE PARAMETERS FOR MEANDERS IN ALLUVIAL VALLEYS

	Meander length to channel width	Amplitude to channel width	Meander length to radius of curvature
Inglis (1949, pt. 1, p. 144, Jefferson data)	$L = 6.6w^{0.99}$	$A = 18.6w^{0.99}$	..
Inglis (1949, pt. 1, p. 149, Bates data)	..	$A = 10.9w^{1.04}$	..
Leopold and Wolman (this paper)	$L = 10.9w^{1.01}$	$A = 2.7w^{1.1}$	$L = 4.7r_m^{0.98}$

meander loops is determined more by erosion characteristics of stream banks and by other local factors than by simple hydraulic principles.

The U. S. Waterways Experiment Station studies (Friedkin, 1945, p. 15) showed that in uniform material amplitude did not progressively increase nor did meander loops cut off as the meanders migrated downstream. Friedkin interpreted this to mean that "natural cut-offs result from local differences in the erodibility of bank materials (p. 16)."

These considerations appear to support the concept that a limit to the amplitude of meanders is provided by the formation of cut-offs. The rate of downstream migration relative to the rate of growth of the amplitude of bends probably depends on the distribution of shear against the stream banks, which in turn is a function of channel shape. The latter is determined at least partly by the characteristics of the flood-plain alluvium and the moving debris.

We believe relations between channel width, meander length, and radius of curvature, in contrast to amplitude, are to a great extent independent of bed and bank materials and are related in some unknown manner to a more general mechanical principle.

Examining the correlations presented above one can write

$$L = 10.9w^{1.01} = 4.7r_m^{0.98}.$$

Assuming the exponents to be unity and solving for the ratio of radius of curvature to width,

$$r_m/w = \frac{10.9}{4.7} = 2.3.$$

Computing the value of r_m/w river by river from the data in the appendix rather than equating the least-square regression lines, it is found that in the sample of 50 rivers the median value is 2.7, mean 3.1, and two-thirds of the cases lie between 1.5 and 4.3. About one quarter of the values lie between 2.0 and 3.0. The tendency for similarity in the ratio of curvature to width makes all rivers look quite similar on planimetric maps, as suggested by Figure 3. In fact when one inspects a planimetric map of a river without first glancing at the map scale it is not immediately obvious whether the river is large or small owing to this tendency for similar ratio of radius to width regardless of river size.

As will be mentioned later, the value of the ratio of radius of curvature to channel width has an important bearing on resistance to flow, and this appears to offer some clue as to the hydraulic basis for the observed geometric similarity among channels of different sizes.

Curves in Sediment-Free Channels

Meandering channels are carved in glacier ice by streams of meltwater flowing on the glacier surface. To obtain measurements of such channels we went to one of the Dinwoody glaciers on the east side of the crest of the Wind River Range in Wyoming (Lat. 43°12′, Long. 109°38″). The meandering channels on this small valley glacier can be seen in Plate 1. Although concentrated primarily upon the steeper slope downstream, meandering channels began on the relatively flat portion of the profile, that is, near the point of inflection from concave to convex portions of the surface profile. The channels extended downstream to the convex-upward part of the glacier snout.

Some of the meandering channels were incised 6 to 12 feet into the ice, and those which appear on Figure 1 of Plate 1 are of this kind. Satisfactory measurements of these incised meanders could not be obtained, but the channel shown in Figure 2 of Plate 1 incised 1 to 2 feet, appeared to be comparable to the more deeply incised streams.

Comparison of the meandering stream with incipient channels being formed by water flowing over the ice surface led us to conclude that an irregular sheet of water gradually concentrated in a shallow flat channel at some place where local factors favored melting. The meandering form developed as the flowing water gradually melted a channel downward. Channels in the ice only a foot wide and 2 inches deep had no well-developed meander form, but another channel of similar width but 1 foot deep showed a well-developed sinusoidal pattern.

Although occasional rocks and pebbles occurred in the channel none were in motion at time of observation. In some bends a few pebbles had collected on the inside of the curve in the position of the point bar in ordinary streams. The surface of the glacier in the locality was strewn with cobbles and rocks in a random manner owing to the fact that the channel measured was near the edge of a poorly developed medial moraine.

In several places terrace remnants were cut into the ice and abandoned as downcutting progressed. No pebbles were seen on these terrace remnants. The meanders appeared to be cut by melting alone with no effective help from abrasion by transported pebbles.

Over-all dimensions (in feet) and flow parameters of the channel pictured in Figure 2 of Plate 1 were as follows:

Channel width	2.3
Channel incision below ice surface	1.0
Width of surface of flowing water	2.0
Mean depth of flowing water, d_m	0.25
Average velocity from measurements of flow, v	5.3 ft/sec.
Average meander length	20
Average channel slope, s	0.023
Discharge	2.4 cfs
Froude number $v/\sqrt{gd_m}$ (where g is the acceleration of gravity)	1.9

The Froude number, 1.9, was well within the range of supercritical or shooting flow. In shooting flow it is usual to observe surface wave trains and the diagonal crisscross of

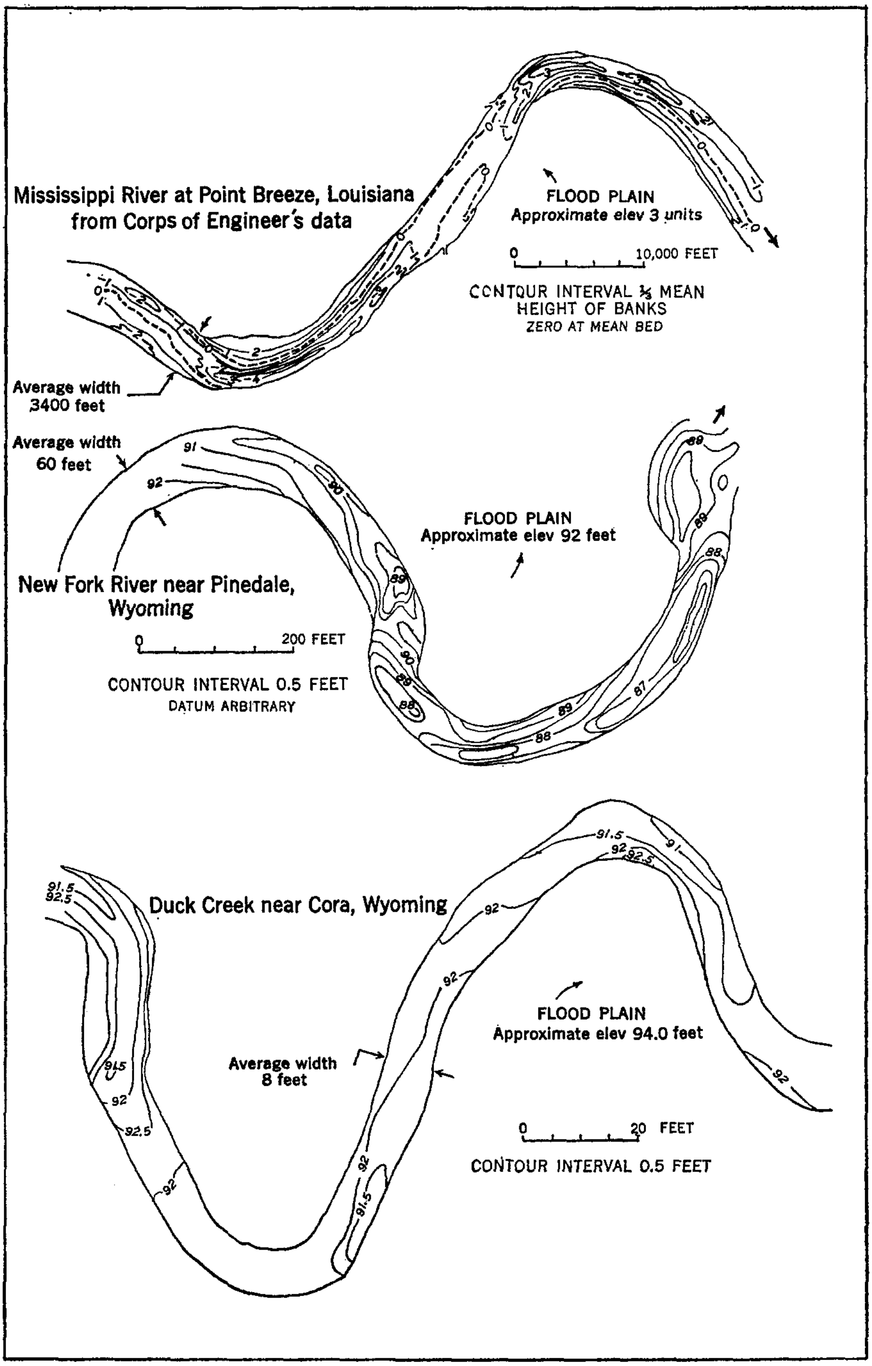

FIGURE 3.—PLANIMETRIC MAP AND BED TOPOGRAPHY OF A MEANDER OF THE MISSISSIPPI RIVER AT POINT BREEZE, LOUISIANA, THE NEW FORK NEAR PINEDALE, WYOMING, AND DUCK CREEK NEAR CORA, WYOMING

Scales are chosen so that meander length is equal on the printed page.

surface waves reflected off the channel sides. If these were present they were masked by the violent turbulence, although they were probably the cause of a marked hump in water-surface elevation extending along the channel center line in some places. The water was conspicuously superelevated on the outside of the curves.

The measurements of meander length and width of this stream cut in ice are plotted on Figure 2. The meander length of channels developed in ice bears the same relation to channel width as in ordinary meandering streams. Measurements made by D. G. Anderson of meanders of small streams on the ice of Chamberlain Glacier, Alaska, confirm this conclusion (Oral communication to Leopold, 1958).

Data on meanderlike phenomena from another source deserve mention as possibly having some relation to the present problem. In the Gulf stream of the North Atlantic oceanographers have found bands of relatively high speed which have a meandering pattern in plan view. Stommel (1954, p. 887) described these as "horizontal eddy and meander formations," and Von Arx (1952, p. 213) used the term "meandering current." These currents appear both in cross sections of the velocity field and in planimetric maps of the temperature field. Maps of the temperature field at a depth of 200 meters have been published by Stommel (1954, Fig. 2) and by Fuglister (1955, chart 3a). From some of these published maps rough measurements of meander length may be obtained and, less satisfactorily, estimates of current width. Better estimates of current width are obtainable from velocity cross sections published by Von Arx (1952, Fig. 2) and Worthington (1954, Fig. 9).

Four measurements of the width and meander length were possible from the published data, and these are plotted on Figure 2. Although they fall slightly below the line drawn through the points representing river data, the graph suggests that meandering currents in the Gulf Stream bear certain analogies to river meanders. Although frictional flow in rivers should not be compared to geostrophic flow of an ocean current, the observed similarity in meander dimensions appears worthy of attention.

The importance of meanders in ice and in ocean currents lies in the fact that both flow systems exhibit meanderlike phenomena in the absence of sediment debris. The establishment of meander length–width relations similar to those of sediment-laden rivers suggests that sediment alters or affects but does not cause the meander pattern. (For a contrasting view *see* Matthes, 1941.)

Channel Cross Sections and Longitudinal Profiles

From a study of bends of the Garonne River, Fargue (1908, p. 25) stated as a general rule of river behavior that the shallowest sections occur downstream from the crossover, and the deepest sections downstream from the axis of the bend or point of maximum curvature. The topographic maps of sample meanders in Figure 3 show in a general way the features described by Fargue and many workers after him. These same figures, however, also indicate that the location of deeps and shallows may be highly variable. For example, data from 25 bends on Buffalo Creek near Buffalo, New York, indicate that the deep points are usually found downstream from the point of maximum curvature but do occur as well upstream from this point (Parsons, 1959). On a reach of the Popo Agie River near Hudson, Wyoming, containing four successive bends of amazing symmetry deeps occurred upstream from the point of maximum curvature and not downstream.

In the reach of the Mississippi River shown in Figure 3 the deepest portion occurs at the axis of curvature in one bend but somewhat downstream from this axis in the other bend. In the New Fork the deepest portion in one bend occurs practically at the point of inflection. The reach on Duck Creek more nearly conforms to Fargue's generalization.

There seems little doubt that the depth of water at a bend of a river is closely correlated

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Plate 1.—MEANDERS ON DINWOODY GLACIER, WYOMING

Figure 1.—In middle foreground are meandering channels of meltwater streams carved in the glacier ice. View southwesterly from Sentinel Peak; photograph by Mark F. Meier

Figure 2.—Channel carries meltwater from glacier. The water flowed at high velocity and was violently agitated.

FIGURE 1

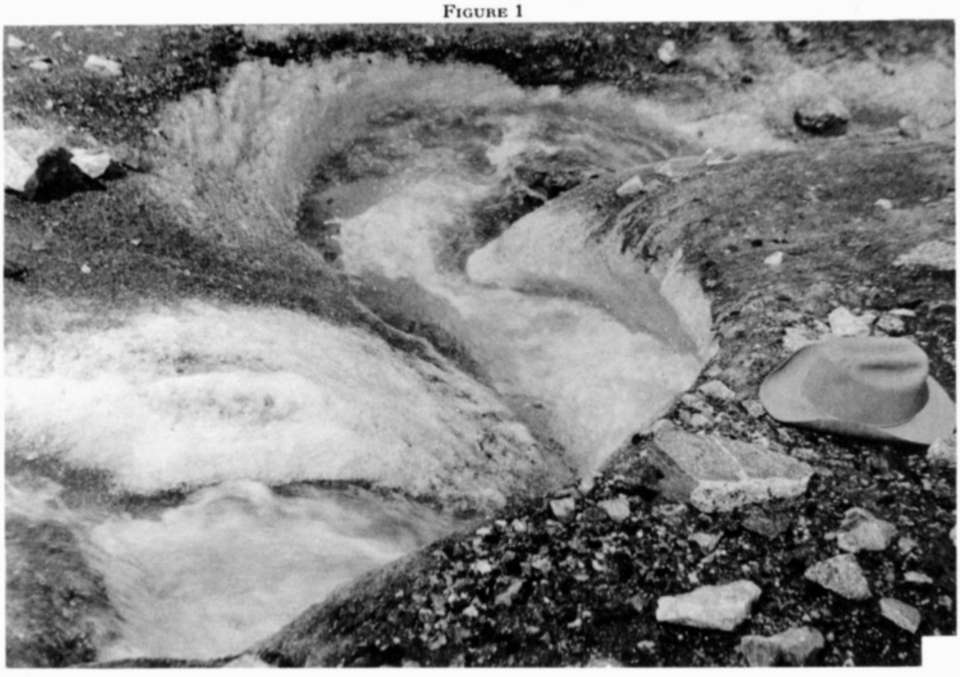

FIGURE 2

MEANDERS ON DINWOODY GLACIER, WYOMING

with the bend curvature. Using 103 measurements of mean depth and corresponding radii of curvature on the River Elbe, Leliavsky (1955, p. 118) shows that the two factors bear a simple linear relation and that depth increases inversely as a function of radius of bend.

The point of inflection in a river curve is closely associated with a shallow portion of the reach, or a depositional bar on the bed. The occurrence of a bar or riffle in the bed of a straight river reach should correspond morphologically therefore to the point of inflection of a meandering channel. It is on this basis that twice the distance between successive riffles in a straight reach was compared with the wave length of meanders (Leopold and Wolman, 1957, p. 55).

Not only do the riffles in straight channels tend to be in a position analogous to comparable shallows in a meandering reach, but there also is some tendency for the shape of the bars to be the same. In some straight reaches successive gravel or sand bars which constitute the riffles assume the form of lobate wedges sloping alternately toward one bank and then the other. In some meanders the bar near the point of inflection slopes sharply across the channel toward the deep near the concave bank downstream. This is apparent on the topographic map of the New Fork (Fig. 3) and is accentuated at low flow when water flows diagonally across the channel down the steepest local bed slope. At high flow there is rapid divergence as the cross-sectional area expands.

The longitudinal profile of bars, whether associated with the point of inflection of a meander, or a riffle in a straight channel, is usually moundlike rather than dunelike and is asymmetric in cross section. Perhaps ideally the bar has a dunelike profile with a short, steep downstream face. The topography of the bend in the Mississippi River (Fig. 3) suggests this, but many are less regular.

The bar is asymmetric in cross section in a different sense on upstream and downstream faces. In a meander, as one moves downstream toward the point of inflection, the surface of the bar slopes laterally toward the concave bank, but immediately having passed the crest of the bar the asymmetry changes, and the surface of the bar slopes toward the opposite bank.

For this reason the channel cross section in a meandering reach is symmetrical only for a very short distance. We estimate that on the average the distance through which the cross section is essentially symmetrical is only about one-tenth wave length. This short distance can be seen on topographic maps of Figure 3.

The importance of the configuration of the bars and their relation to the deeps lies in the relation of bed topography to bed shear. River pilots and others know that in large rivers the crossings (points of inflection) tend to scour at low flow and fill at high flow; the bends (pools) tend to fill at low flow and scour at high (Straub, 1942, p. 617; Lane and Borland, 1954, p. 1075).

Local bed shear must be greater at points of scour than where filling occurs, but direct measurements of bed shear are few, and little is known in detail about the relation of bed topography to stress on the bed.

Lane and Borland (1954, p. 1079) explain the observed sequence of scour and fill as a consequence of the different shape of cross section in pool and riffle. They say that the crossing or riffle has a larger cross-sectional area than the pool during high stages and a smaller area during low stages. Laursen and Toch (1954, p. 1085) state that width often tends to be greater at a crossing than at a bend. The expansion in width they believe would cause deposition on the wide crossing while scour occurs in the narrower bend.

We have made measurements of slope and of depth over pool and riffle separately at various stages of flow in an attempt to compute approximately the variation of mean stress. At low flow the bend is a still pool over which the water has a relatively flat slope. In contrast, in the riffle the water tends to plunge over the steep downstream face of the bar. As stage increases, the slope over the pool or bend increases, while that over the bar decreases until near bankfull stage when the water surface attains a continuous uniform slope, and all obvious surface effects of the shallow bar have been erased.

With increasing stage, depth increases in both bend and crossing. The product of depth and slope, then, increases with stage more rapidly in pool than at the riffle or crossing.

In the one stream measured by us, Seneca Creek near Dawsonville, Maryland, the computed mean shear is greater in the riffle at low flow and greater in the pool at high flow (Leopold, in press). This observation is in qualitative agreement, then, with the concept of scouring in the crossing at low flow and in the bend at high flow. The analysis is incomplete, however, inasmuch as variations in the distribution of velocity in successive cross sections have not been included in the computation of shear, and additional observations

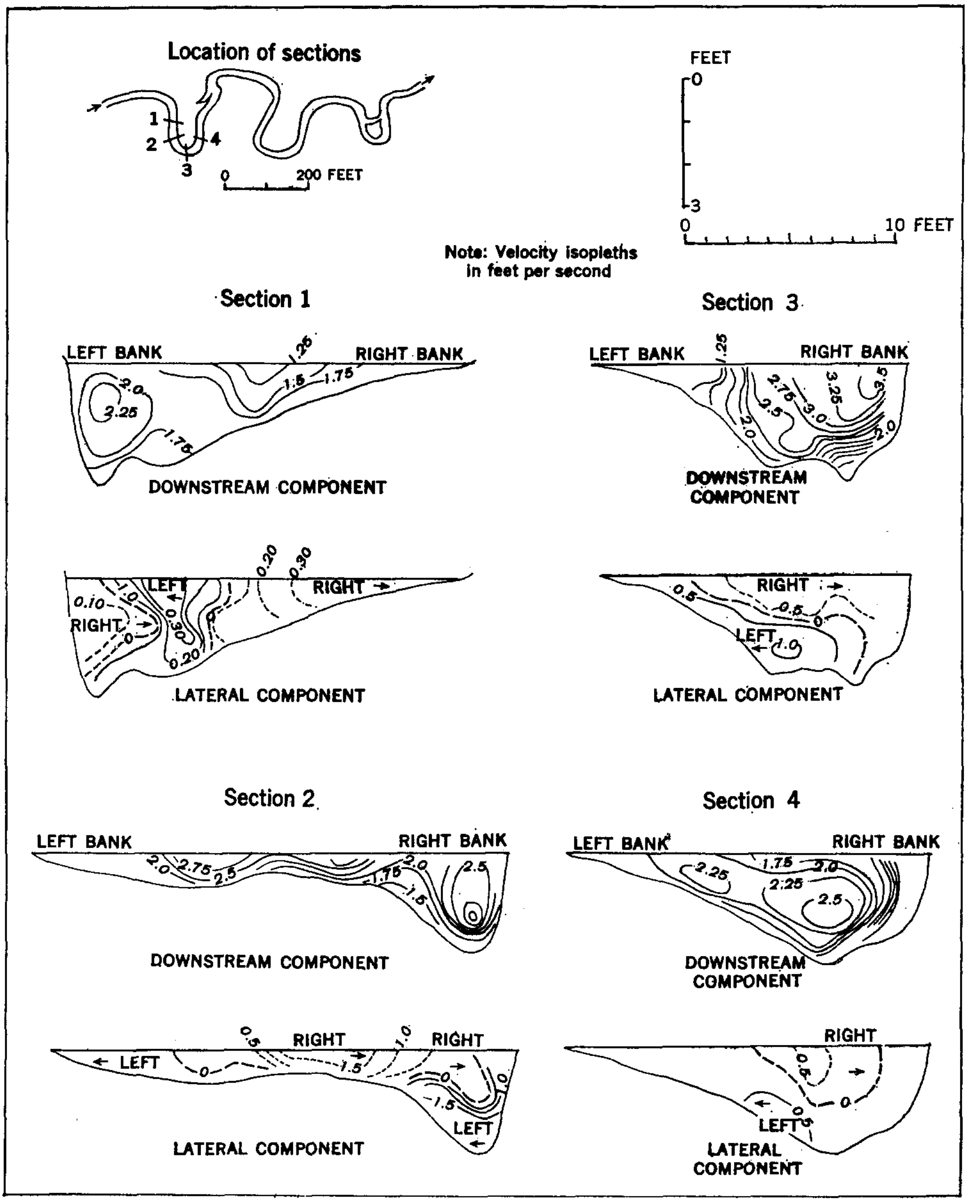

FIGURE 4.—LATERAL AND DOWNSTREAM COMPONENTS OF VELOCITY AT VARIOUS CROSS SECTIONS IN A BEND, BALDWIN CREEK, NEAR LANDER, WYOMING

elsewhere are necessary before any generalization can be made. Mean depth and slope, even over a short reach, probably do not measure the true shear exerted on any local area of bed. Direct measurement of shear stress in various parts of a bend at various stages is needed.

DYNAMIC AND FLOW CHARACTERISTICS

Pattern of Flow in a Meander Bend

Relatively few detailed measurements of the distribution of flow in a natural meander have been published (Blue *et al.*, 1934; Eakin, 1935; Leliavsky, 1955, p. 96–100; Van Til and Tops, 1953, p. 19–23). The velocity distributions which follow are intended both as additions to the slim store of data and as illustrations of the general characteristics of flow in bends in natural channels. Figure 4 shows downstream and transverse components of velocity at the axis of the bend on Baldwin Creek near Lander, Wyoming. Point velocities were measured by Price current meter and horizontal angles by a vane attached to the wading rod. Measured angles were used in conjunction with the measured velocity to compute the lateral components of velocity. Measurements were made at a high but less than bankfull stage.

At least in narrow channels a cross-channel velocity component is directed toward the convex bank or point bar near the bed, and toward the concave bank near the surface. Continuity requires, then, that surface water plunge downward near the concave bank and that some bed water emerge at the surface near the convex bank.

This circulatory motion in the cross-sectional plane of a channel, as clearly explained by Thomsen (1879), results from the larger centrifugal force on fast-moving surface parcels than on slower-moving ones near the bed. The motion gives to an individual water parcel a path resembling a helix.

Despite the tendency for circulatory motion in the stream cross section, surface stream lines (Fig. 5) show that no single parcel of water crosses completely from the convex to the concave bank. The maximum cross-channel motion does not exceed perhaps two-thirds of the channel width in any given meander bend. This accounts for the observation by Friedkin (1945, p. 5) in laboratory models that material eroded from one bank tends to deposit on a point bar downstream on the same side of the stream.

Matthes (1941, p. 634) stated that in wide, shallow rivers helical circulation does not exist. It is our opinion that in rivers of large width/depth ratio the helical motion exists, but not as a single rotating cell involving the whole channel (Nemenyi, 1946). The existence of several cells of circulation in the Tamm Bend data from the Mississippi can be seen in the distribution of the lateral components of velocity published by Eakin (1935). As in the atmosphere, circulation cells of large magnitude tend to break down into smaller cells. The "polar-front" cell and the "trade-wind" cell of Rossby (1941, p. 610) are directly driven by dynamic forces which we consider analogous to the direct helical motion in river bends due to the combined effect of frictional and centrifugal forces. Rossby's "frictionally driven" circulation cell in the middle latitudes furnishes, in our opinion, an analogy to those circulation cells seen in the Eakin data of the Mississippi which rotate in a direction opposite to the helical motion expected in a single cell occupying the entire cross section.

It is well documented that the water surface is superelevated near the concave bank of a channel bend (Blue *et al.*, 1934; Eakin, 1935; Leliavsky, 1955, p. 123; and others). The amount of the superelevation is proportional to

$$\frac{v^2 w}{g r_m}$$

where v is the mean velocity, w is channel width, r_m the mean radius of curvature, and g the acceleration of gravity (Leliavsky, 1955, p. 122).

This superelevation is a consequence of the curved path of water flowing around a bend and is a correlative of the helical motion.

Generalized Picture of Flow in a Meander

Measurements in meandering streams and in curved flumes (for example Mockmore, 1944, p. 569) allow the construction of a generalized picture of the flow pattern in a meander (Fig. 5). The isometric view of the two principal components of velocity at various positions in the bend show the main features. The scale is such that superelevation of the water surface in the bend does not show on the diagrams but is implied by the velocity distribution.

At the crossover or point of inflection of the bend (Fig. 5, section 1), the cross-sectional shape is not completely symmetrical but is slightly deeper near the bank which was concave in the bend immediately upstream. Downstream from the crossover the section (2) becomes approximately symmetrical, but, because this section is in the bend, some cross-current component exists in accord with the curving stream lines.

The velocity in a meander crossover is not symmetrically distributed. As would be expected, proceeding downstream from the axis of the bend the thread of maximum velocity is much closer to the concave bank than to the center of the channel. The high velocity, moreover, continues to hug this side through the point of inflection of the curve.

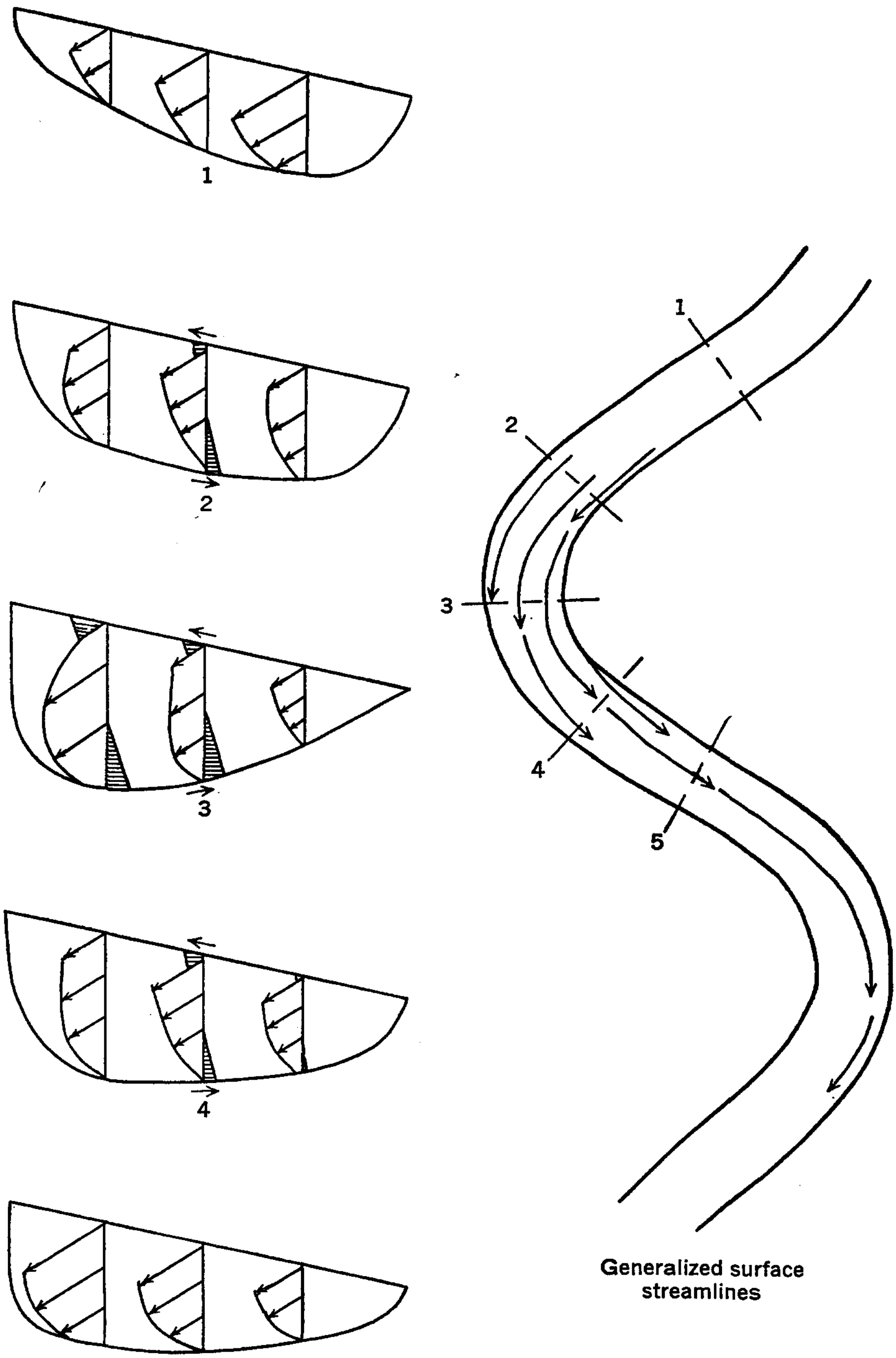

FIGURE 5.—ISOMETRIC VIEW OF GENERALIZED DIAGRAM OF FLOW DISTRIBUTION IN A MEANDER

Showing downstream (open parabolas with arrows) and lateral (closely lined areas) components of velocity as vectors, and surface stream lines. All sections viewed from a changing position to the left of and above the individual section.

The asymmetry in the velocity field persists downstream from the point of reversal of curvature, at least as far as the most nearly rectangular and symmetrical channel cross section.

Downstream from the symmetrical cross section the maximum downstream velocity approaches the concave bank. The highest velocity at any point occurs near the concave bank just downstream from the axis of bend. Individual filaments of water accelerate and decelerate along a stream line. The maximum velocity is not always associated with the same parcels of water.

Measured distributions of velocity show also that the maximum-point velocity in the bend is depressed below the water surface. These observations agree with the measurements made by Mockmore (1944, p. 596) in a curved flume.

An effect of increasing discharge in a meander reach is to move downstream the position on the concave bank where the thread of maximum velocity impinges. The stream lines of highest velocity do not touch the concave bank but approach it and then swerve into a course more or less parallel to that bank.

Channel Shape and Movement in Relation to the Flow Pattern

The slight lack of congruence of stream-line curvature with bank curvature leads to the tendency for the locus of point-bar deposition to occur downstream from the axis of the bend. A building point bar tends to concentrate caving of the bank downstream from the axis of bend and thus accounts for the progressive down-valley movement of meander bends. This same tendency exists in all river curves, even in those lacking sufficient symmetry to be called meanders, and this accounts in part for normal movement of river channels and the consequent construction of flood plains.

Material slumping into the bed by bank caving is caught in the transverse component and carried toward the middle of the stream. If the location from which it is derived is far enough downstream in the bend, insufficient length of reach remains for the material to get all the way across the channel. Instead it moves through the crossover reach without having crossed the channel. Once into the reversed curve the particle moving near the bed is drawn toward the same bank from which it started rather than to the opposite bank. This does not rule out the likelihood that much fine material which is not carried near the bed of the stream will be deposited at a downstream location on the side opposite to the one from which it was derived.

The plunging or diving motion of surface water near the concave bank is one factor tending to depress the point of maximum velocity below the surface near the axis of the bend. Bank friction also has a similar effect. Because the highest velocity gradient on both bed and bank occurs where the thread of highest velocity is nearest the boundary, this maximum velocity gradient is equivalent to maximum shear stress. Its location accounts, then, for deepening of the bend reach near the concave bank. This scour, together with building of the point bar, explains the triangular shape of the cross section.

Increasing the depth of water in the channel reduces the vertical velocity gradient. In a channel that is very deep relative to its width, the effect of bed friction becomes relatively small, and the velocity is nearly constant with depth except close to the bed. Thus the helical circulation becomes negligible. Flow in a curved channel of great depth and small width approaches potential flow—that is, the downstream velocity varies across the channel inversely as the radius of curvature of the stream lines. This was demonstrated by Wattendorf (1935, p. 574) in a deep, narrow, curved flume. In the absence of the circulation in cross section the thread of high velocity hugs the convex bank and does not cross the channel. Thus the mechanism for building the point bar is probably absent in channels of extremely small width–depth ratio.

Effect of Flow Pattern on Deposition and Erosion

The vigorous cross currents near the bed can transport considerable quantities of bed material from the concave to the convex side of the channel. On a meander bend of Baldwin Creek (Fig. 4) depth-integrated samples of suspended load were taken at each of five verticals spaced across the stream and repeated in each of five cross sections along a meander wave. A nearly uniform concentration of sediment existed across the section in a crossover. In the sections located in the curved reach the concentration increased markedly near the convex bank.[1] A similar distribution of sediment was observed by Eakin (1935,

[1] Dr. John P. Miller assisted the senior author in obtaining these measurements.

p. 471) in his samples taken at Tamm Bend on the Mississippi River. This increased concentration near the convex bank is attributed to the fact that water in this location has, to some extent at least, come laterally in a path close to the bed and is comprised, therefore, of water elements with relatively high concentrations of sediment.

Although there is some evidence that rifling in a spirally welded pipe tends to increase the ability of a flow to transport sediment over that of a cast-iron pipe (M. L. Albertson, oral communication to authors), evidence from preliminary experiments (L. M. Brush, oral communication to Wolman) indicates that transport in a meandering channel is much reduced over that in a straight channel of similar dimensions and boundary roughness. This effect of curvature on transport is in accord with the added energy losses accompanying increased curvature.

The cross-channel component of bed flow toward the convex bank is probably the principal mechanism for the building of a point bar. The path of particles which are deposited on a building point bar has, of course, only a small component across the stream compared with a large down-stream component. The lobate form of a point bar may be compared to a sector of a very flat cone. A bed particle, taking a path along a chord of this cone, may climb the inclined plane or ramp presented by the cone. Thus, even some of the coarser debris of the channel bed will be deposited on the point bar at an elevation considerably higher than the average level of the stream bed.

Variation in discharge would lead one to expect that a thin layer of coarse material may be deposited on the point bar at one time and at another time a layer of fine material may be deposited upon the coarse. On the average, fine material will be carried higher up the bar than will coarse material, so the point bar would show on the average a rough gradation of size from coarse near the base to fine near the flood-plain level. In many instances, then, a growing point bar would not merely lap over the coarse material on the stream bed.

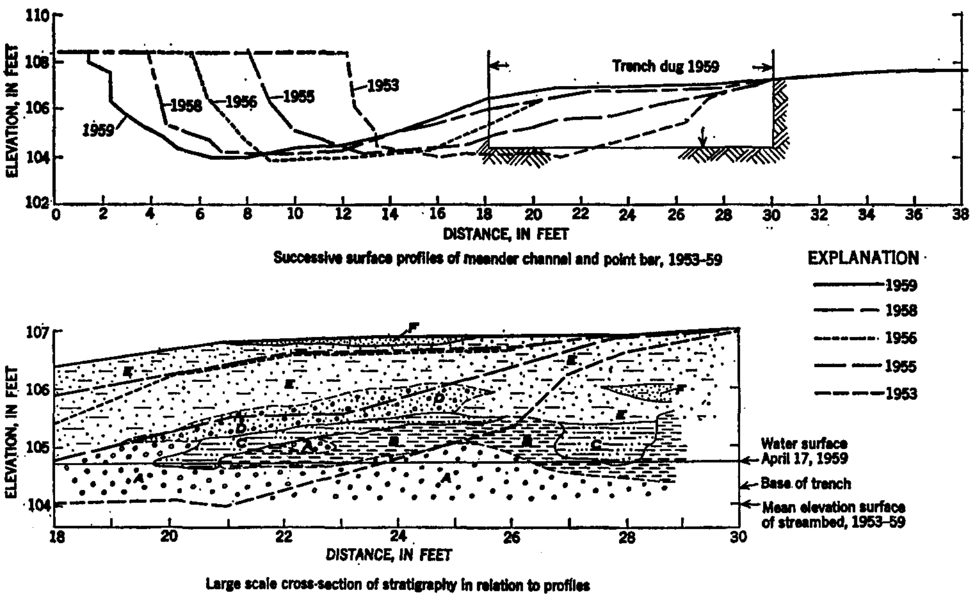

FIGURE 6.—STRATIGRAPHY OF POINT BAR IN RELATION TO MEASURED CROSS-SECTIONAL PROFILES DURING SIX YEARS OF OBSERVATION, WATTS BRANCH, 1 MILE NORTHWEST OF ROCKVILLE, MARYLAND

Key to stratigraphic units is as follows:
A Gravel, mostly 3–8 mm with considerable 8–20 mm and a few 64 mm
B Olive-gray clayey silt, with organic matter, small mica flakes
C Orange-brown, mottled sandy silt with some clay, lenses of leaflike organic mterial, mica
D Coarse sand, brown-stained, with pebbles up to 8 mm, some fine roots *in situ*, lenses of silt
E Brown sandy silt gradually changing upward in places into fine sand
F Fine sand with some silt

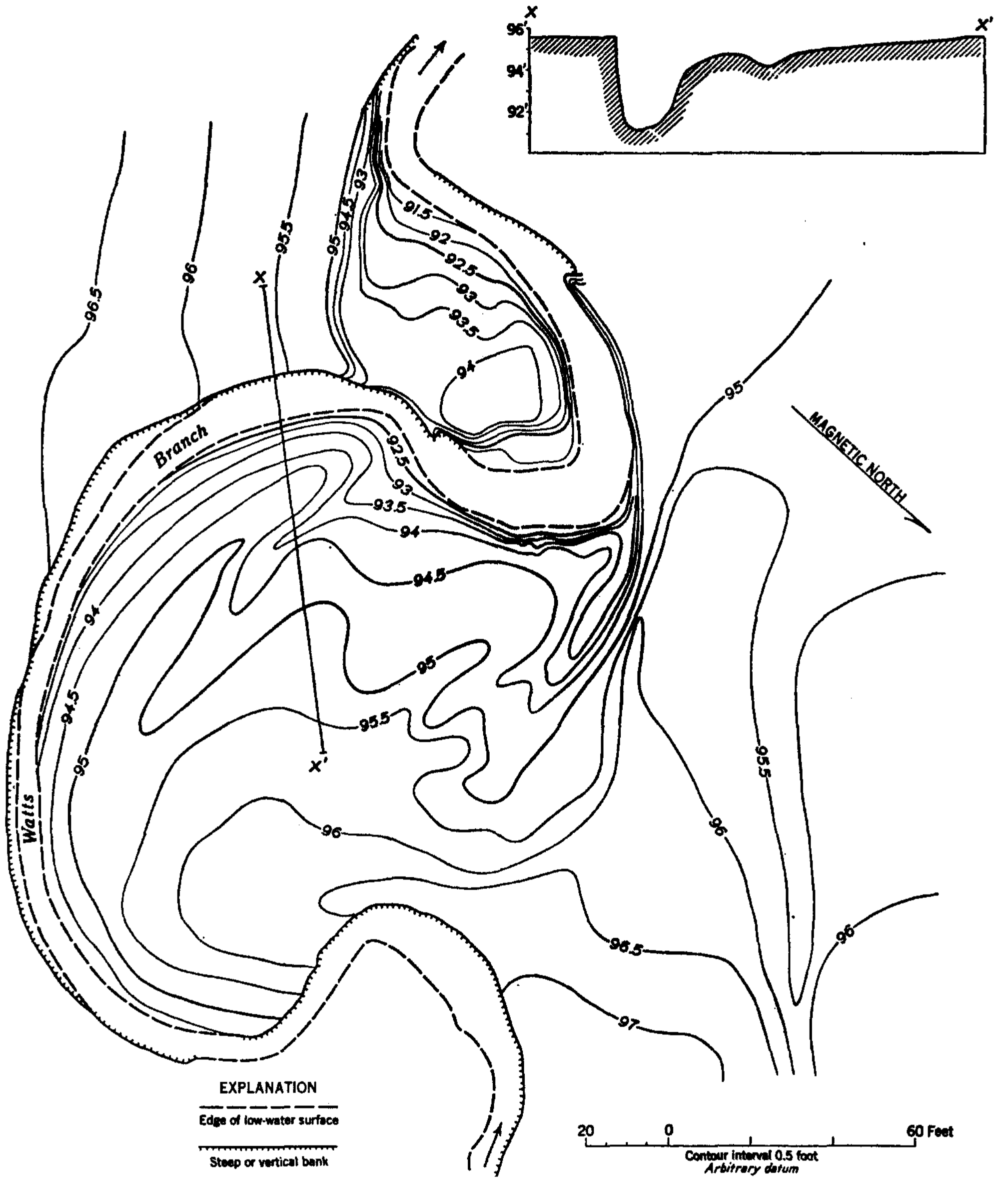

FIGURE 7.—MAP AND CROSS SECTION OF A TYPICAL POINT BAR OF THE FLOOD PLAIN OF WATTS BRANCH, 1 MILE NORTHWEST OF ROCKVILLE, MARYLAND
After Wolman and Leopold (1957)

An unusual opportunity to verify this reasoning was provided by a series of cross sections made in a meandering reach of Watts Branch near Rockville, Maryland. During 6 years of observation the stream has moved laterally a distance equal to one channel width. At the end of this period of observation a trench was dug in the point bar normal to the channel. The stratigraphy revealed in this trench superimposed on known surface positions is sketched in Figure 6. This trench was located on the right side of the stream on section x-x′ of Figure 7.

Approximate contact surfaces between materials of different textures are more or less parallel to past surface profiles. Coarse material characteristic of the bed is observed in a position well above the channel bed. In addition, there is no doubt that deposition occurred up to the level of the water surface at bankfull stage. In the same period deposition over the flat flood plain by overbank flow has been too small to measure. These observations appear to confirm the authors' (Wolman and Leopoldt 1957) hypothesis that point-bar building i, the primary process of flood-plain developmens in flood plains of this type.

Bank erosion is greatly influenced by wetting of the bank materials. Arroyos cut in fine-grained alluvium experience most bank cutting after, not during, flow. The wetting causes later slumping (Leopold and Miller, 1956, p. 5). Bank erosion is also enhanced by return seepage of water which infiltrated the banks during high flow. Upon lowering the stage the balancing pressure of the water in the channel is released, and the banks slump or collapse (Inglis, 1949, pt. 1, p. 152). A study of bank cutting in Watts Branch near Rockville, Maryland, showed that a combination of bank wetting and ice-crystal formation promoted the greatest bank erosion (Wolman, 1959, p. 214). Although the largest discharges occurred in summer, the winter provided more thorough soil wetting which, in combination with freeze and thaw, led to maximum bank erosion.

As bank erosion occurs in the bend of a meander, over a period of time it is usual for an approximately equal amount of deposition to occur on the opposite bank. This general equality of deposition and erosion is the reason width and cross-sectional area remain about the same as the channel moves laterally across the flood plain.

Meander Mechanics and Physiographic Problems

General Discussion

The vagaries of nature provide endless opportunities for perturbations in the flow—local bank erosion, chance emplacement of a boulder, fallen trees, or blocks of other vegetation—any one of which would alter the path of a straight channel. Thus one need hardly inquire why a stream channel is not straight. On the other hand, a random succession of chance perturbations might be expected merely to result in random bends of different patterns. Although this situation describes many channels, the existence of beautifully symmetrical meander bends and the remarkable similarity of bends in rivers of different sizes and physiographic settings must be explained. There is not yet available any theory or dynamical principle which explains qualitatively the characteristic patterns common to meandering channels. In the absence of such a general principle, however, attempts have been made to explain at least qualitatively how symmetrical successive bends begin and grow, and how size of bends is related to the size of the river.

Initiation and Development of Meanders

A large body of experience and literature on river regulation has been built up by European engineers on rivers in Europe, India, and Africa. This experience has covered a far greater span of time and of field conditions than has American practice. Regarding meanders, Leliavsky recently summarized the salient concepts developed (1955, esp. p. 111–141). Apparently the consensus of these workers is that the effect of helical flow is the dominant factor. Leliavsky expresses it (p. 128) as follows:

> "For some reason or other, a small abrasion ... in the ... bank of a straight channel is supposed to have taken place. The water particles moving alongside the eroded portion of the bank follow a curved trajectory and develop, consequently, a centrifugal force. This force, in turn, gives rise to a local helicoidal current, which intensifies the original abrasion and works its way deeper and farther into the shore, ... until the whole channel becomes finally involved in the process and the eroded material accumulates on the opposite bank. This, then, is the birth of a meander."

Prus-Chacinski (1954) also argues that helical flow is the basic mechanism leading to meandering. He showed, further, that, by introducing an artificial secondary flow at the entry to the first bend, it is possible to produce various kinds of secondary circulations in the next successive bend which, in turn, affect the circulation in the next bend, and so on. He demonstrates that the downstream effects of a given circulation pattern are quite persistent, often through several successive bends. The "cause" of meandering Prus-Chacinski ascribes to any disturbance which produces an initial secondary circulation.

It seems clear that helical flow may play an important role in the process of deposition on a point bar. A building point bar helps concentrate shear against the concave bank and thus promotes bank caving and channel movement. Even denying that helical flow exists in wide rivers, Matthes' concept of meander development is closely allied with that just mentioned.

Matthes (1941, p. 633) indicates that bank cutting and orderly transfer of sediment to its place of deposition on point bars is a principal requirement for meandering. He observed that material tends to be deposited on point bars on the same side of the channel as that of the concave bank from which it was eroded. Thus, by bank erosion of a concave bank and concurrent deposition on a point bar across the channel, the channel will move laterally and downstream. Thus, the meander wave may migrate downstream generally maintaining its configuration if the materials are uniform. Friedkin's concept of the process (1945, p. 4) is essentially similar to that of Matthes (1941).

Only a few observers have studied this process of meander formation and development in laboratory channels. Quraishy (1944) described the development of a sinuous channel from an initially straight one in a sand-bed flume. Of particular interest was the formation of a series of small dunelike ripples of sand which he called "skew shoals." These developed on the initially flat bed after the sand grains had been in motion for a short time. These skew shoals were regularly spaced and alternated in position along the channel sides. After their full development they obstructed the path of water and forced it to assume a sinuous course.

This kind of mechanism is highly suggestive because it produced alternating bends, spaced uniformly along an initially straight channel at distances depending on the spacing of depositional barlike features on the bed. Although the Quraishy experiments do not indicate how the shear force of flowing water interacts quantitatively with bed debris to govern size and spacing of the dunelike features, it implies that the geometry of meander waves might be a function of bed forms.

The coincidence of the spacing of pools and riffles in straight reaches and meander lengths in rivers of comparable size might well be the result of a principle of bar or dune formation, as yet unknown, allied in some manner to skew shoal formation. Wolman and Brush (in press) found, however, that in noncohesive sands similar skew shoals formed only at supercritical flow (Froude number, based on mean depth and velocity, greater than 1.0). Although their wide shallow channels did not meander, Friedkin (1945, p. 4) produced a meandering channel from an initially straight one and concluded that bank erosion alone was sufficient to initiate meandering as long as channel widening and shallowing did not proceed too rapidly. In the study by Wolman and Brush (in press) helical flow was observed in the channel bends produced from the skew shoals.

Werner (1951) among others[2] has attempted to develop a general equation to describe meander formation and geometry. He (1951, p. 899) has suggested that the initiation of meanders is caused by "some local impulse or disturbance" which creates a transverse oscillation in the straight stream channel. Expressing velocity in terms of roughness, slope, and depth, as given by the Manning equation (p. 900), he derives an equation in which meander length is a function of initial width, channel roughness, slope, depth, and a coefficient proportional to sediment load.

An assumption is made that sediment prolongs the period of oscillation, inasmuch as any mechanism relying principally on inertial oscillations of the water within the channel width (seiche effects) gives wave periods much too short to account for meanders. The equation agrees qualitatively with respect to the interrelationships of some parameters. It indicates, for example, that meandering will be inhibited at high slopes, a conclusion which appears to be borne out in natural rivres (Leopold and Wolman, 1957, Fig. 46). On the other hand, the equation does not appear to describe quantitatively the observed relation of meander length to channel width. Further, if sediment load is not a necessary condition of meandering, the equation is invalid, for, when the load is zero, meander length is zero, and hence meandering should not occur.

Hjülstrom (1942, p. 252; 1949, p. 84) presented an expression for meander length based on seiche theory. He computed wave length as a product of seiche period and wave celerity, period being that of a seiche having a fetch equal to the width of the meander belt and water depth equal to that of the river. Wave celerity he considered to be the mean down-valley velocity of the river. The wave length so determined was also made a function of a coefficient of turbulent friction.

The resulting expression provided Hjülstrom a basis for reasoning about the effects of changing discharge on wave length through the interacting effects of water depth, velocity,

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[2] The Coriolis force or the lateral deflection due to the rotation of the earth has frequently been cited as the cause of meandering, but calculations by Ludin (1926) and others indicate that the virtual radius of curvature attributable to the Coriolis force in a stream with a velocity of 3 feet per second would be on the order of 8 miles at a latitude of 60°.

and turbulence, and his argument showed keen insight and understanding of field conditions (1949, p. 86–88). But the formula has the disadvantage of making meander length dependent on wave amplitude; in our opinion, measurement data do not demonstrate this.

Although a simple comprehensive expression is still wanting, it appears that the forces determined by the velocity distribution, including the helical circulation, are all that is necessary to account for (1) the shape of the cross section in a meander, (2) the depositional and erosional pattern, and (3) the progressive down-valley migration of the meander. These observations are not new, but it is important to emphasize the following idea. Although point-bar formation and associated erosion of the opposite bank are necessary if a straight channel is to develop curves, the concept of helical flow, as Leliavsky (1955, p. 128) recognized, does not seem to explain how the secondary circulation determines the characteristic dimensions or proportions of meandering channels. The existence of meanders on glacier ice also implies that erosion and deposition may be a collateral, not the governing principle of meander development and movement. Because the hydraulic or mechanical significance of the pattern of curvature is closely tied to the fundamental physiographic questions, we consider this aspect of meander mechanics in the following section.

Problem of Channel Equilibrium

The preceding discussion was concerned principally with the initiation and development of a meander, elements that can be discussed in terms of a short reach of river or a single meander wave. There are a host of broader problems of channel adjustment to external controls which might be thought of as physiographic problems for want of a more specific term.

To begin, one might ask how meandering of a channel relates to the fundamental process of stream adjustment and stream equilibrium. It is generally believed that channel equilibrium is constantly approached, although rarely attained, by a process of continual adjustment. To use the words of Rubey (1952, p. 129),

> "....with changing conditions, the stream is constantly cutting or filling and modifying its slope, velocity and cross section so as eventually to accomplish the imposed work with the least expenditure of energy."

If, indeed, a principle of least work is involved—for it is not yet proven—how does the development of a meandering pattern help accomplish this objective? Is the reduction of gradient achieved by increasing length relative to a straight channel necessary because of an excess of energy? Or, as was once believed, is meandering the aimless wandering of a channel too sluggish to accomplish any work of erosion? Does a river reach a stage at which vertical erosion is negligible and thence expend its excess energy on lateral erosion by meandering?

Under what changes of conditions would a river change its pattern from meandering to nonmeandering or vice versa? What would be the effect of an increase or decrease in discharge or in sediment load from the drainage basin?

Opinions on some of these questions have been published, but data or measurements are meager. Existing data may answer a particular question but do not explain why the observed result was obtained. In the following paragraphs some of these questions are considered along with related observations from the literature. Results of recent work, where applicable, are cited, and we suggest what appears to us some of the directions in which future work is needed.

Many of these queries relate to a single, fundamental question—what has the meandering pattern to do with energy expenditure? Inglis (1949, p. 158) states that,

> "Meandering is Nature's way of damping out excess energy during a wide range of varying flow conditions, the pattern depending on the grade of material, the relation between discharge and charge (load) and the rate of change of discharge and charge."

Schoklitsch (1937, p. 149) earlier stated what appears to be the same idea, that meander formation

> "might be due to the fact that the slope in such stretches is too great and is not in equilibrium with the size of the bed-sediment grains."

Water-surface slope of a river is a measure of energy expenditure. Whereas the equilibrium slope is closely related to the size of particles on the bed, there are other factors involved. Hack (1957, p. 61) confirms quantitatively the generally held belief that channel slope is controlled to a great extent by bed grain size and discharge, but to what extent channel shape enters is still unclear. As Rubey explains

(1952, p. 131), channel shape may adjust jointly with slope. At constant discharge if the principal effect of bed particle size is on channel roughness or frictional drag, then other forms of drag need to be considered. Total channel resistance is materially influenced by form drag of various kinds, pools and riffles, bars and dunes, and channel curvature.

There is reason from the hydraulic standpoint to believe that meandering may in part be a function of frictional drag and thence energy loss. More energy loss occurs per unit of length in a curved than in a straight channel of the same depth and cross section, owing to eddying, secondary circulation, or increased rate of shear. These eddy losses result from deflection of the water to a new direction as it moves around a curve or bulge. It is known from hydraulic experiments in pipes (King, 1954, Fig. 87) that energy loss first decreases and then increases with a decrease in the ratio radius of curvature/pipe diameter. Although any curved channel offers greater resistance to flow than a straight one, the minimum increase in resistance is about 40 per cent, and this applies within a narrow range of the ratio of radius of curvature/diameter when that ratio has a value of 2 to 3.

It was pointed out earlier that in meandering channels the comparable ratio, radius of curvature/channel width, is relatively conservative. Values of this ratio also tend to fall in the range of 2 to 3.

Bagnold (in press) suggests an explanation of this minimum increase in resistance at this value. He postulates that, as radius of curvature is reduced and becomes about 2 to 3 times the channel width, an eddy or zone of reverse flow forms just downstream from the bulge or convex bank. With the appearance of the reverse eddy the local width is constricted, and there is a local increase in effective radius of curvature and a net decrease in energy loss.

To the extent that further work confirms the indication that the modal value of this ratio is in the range 2–3, meanders tend to be characterized by a geometric pattern which happens to offer the smallest energy loss of any configuration of curved channel. The significance of this observation is unknown, but it suggests that some principle related to energy conservation does operate in the meander mechanism.

Assuming that a bend actually does tend to develop a configuration such that the energy loss due to the bend is a minimum, it does not follow that the total energy expenditure has been minimized. When a given discharge falls through some specified vertical distance, a certain amount of energy is transferred from potential form to some other form. If the water does not accelerate (velocity remains about constant downstream), then this potential energy is expended as work or heat. The same amount of energy is spent whether the water moves in a straight channel on a steep slope or in a longer curved channel at a smaller gradient. The energy expenditure per foot of channel length is smaller in the longer curved channel than in the shorter straight one. The question, then, is how much energy is utilized per foot of channel length and in what form it is used.

This energy may be spent in moving particles of debris, or it is otherwise dissipated into heat. The energy may be spent in removing particles from the bank and transporting them (bank erosion) or in transporting bed or suspended particles. If this energy is concentrated in such a manner that more of it goes into moving particles from one place than from another, then local scour will occur there and deposition elsewhere. For a channel to be in equilibrium scour must balance fill within the reach in question, and, further, the energy must be so expended that the net amount of debris coming into the reach must equal the net amount carried out of the reach.

It is generally agreed that meandering channels are often stable or in quasi-equilibrium. They may be so even though, over a period of time, a meander wave moves gradually downstream. The slope, discharge, and channel shape tend to become adjusted so that the above requirements are fulfilled. Adjustments in channel shape occur through erosion or deposition which in turn affect velocity, depth, and width. Specific hydraulic requirements relating depth, slope, velocity, and total resistance, including resistance offered by bed configuration (form resistance), bed and bank grains (skin resistance), and channel curvature (a particular kind of form resistance) must also be maintained.

Meandering is one way in which erosion and deposition may change the distribution, location, and amount of energy expenditure per unit of channel boundary. By lengthening the channel between two points at different elevations, the energy expenditure per foot of length is reduced. By bank erosion, point-bar building, and by scour and fill, the channel cross section is adjusted, and the energy expenditure is redistributed. Presumably abrupt discontinuities in the rate of energy expenditure in a reach of channel are less compatible with conditions of equilibrium than is a more or less continuous or uniform rate of energy loss. It may well be that a meandering channel is most stable when the energy loss due to curvature is at a minimum. Such a conclusion is perhaps implied in the modal distribution of values of the ratio of radius of curvature to width in natural streams.

If we view curvature as simply one method of altering the distribution of energy expenditure in a given length of channel, it is clear that the pattern of meandering will respond to changes in discharge and load. It is well known that an increase in discharge in a meandering channel will increase the channel width and will increase the size of the meander bends. Decrease in discharge will gradually reverse the process.

In the natural rivers, geologic and stratigraphic evidence clearly demonstrates that during late Pleistocene time an increasing discharge markedly decreased the gradient of Öster-Dal River and enlarged the width of channel as well as the size of meander beds (Wenner and Lannerbro, 1952, p. 108). During this gradual degradation of the valley floor the meandering pattern persisted. Thus the meander pattern where it exists in nature appears to be a persistent attribute of the river.

Change in load will cause aggradation or degradation and thus change in channel slope and size of bend (Friedkin, 1945, p. 7–9); an increase in slope will produce an increase in meander length and amplitude. Schoklitsch (Shulits, 1935, p. 644–646) and Bagnold (1960) have postulated that at high discharges sediment transport is a function of the rate of work done per foot of channel length, or power intensity. In a meandering channel in equilibrium increasing tightness of bend (curvature) through its effect on the rate of energy expenditure decreases the rate of transport. For equilibrium, then, a balance must be maintained between curvature and transport quite apart from any change in intensity of energy loss brought about by a change in length.

Direction for Future Work

No wholly adequate explanation of meandering is yet available. Probably no single simple mechanism will suffice to explain all aspects of meandering. Although there is general agreement on the manner in which bank erosion and point-bar formation are related to the orderly transfer of sediment which is basic to meandering, no physical or mechanical principle has been identified which explains qualitatively the size and geometry of meander curves.

There is need to investigate oscillatory forces which might explain more adequately the manner in which an initial bulge or depression in a stream bank leads to a symmetrical reversal of curvature.

Although the available velocity distributions in channel bends do permit general descriptions of the flow, many characteristics, including orientation and position of the helical patterns, are as yet poorly defined. More detailed measurements in natural channels are required to define the loci of energy losses and their relation to flow resistance and localized erosion of bed and banks.

The way in which a natural channel distributes the energy loss as between boundary friction, form resistance, curvature, and transport is little understood. Without such understanding it is virtually impossible to explain or predict the behavior of a meandering channel. Although some studies of energy losses in curved channels have been made (Allen, 1939; Leopold *et al.*, in press), it appears to us that laboratory as well as field studies on the distribution of energy expenditure in straight and curved alluvial channels are needed. It would be particularly desirable to map the distribution and magnitude of boundary shear in bends of different patterns but with similar cross section and depth. These observations must in turn be related to the mechanics of sediment transport in meandering channels of different patterns.

The principal unsolved problem with regard to the pattern of flow and its relation to erosion and deposition is in the area of the mechanics of sediment transport. Present theories are inadequate to explain the transport of heterogeneous sizes under the variety of conditions found in nature. The stress needed to induce and maintain motion is probably different for scattered rocks on a sand bed, uniform grading from sand to cobbles, graded cobbles and boulders without sand, or a few isolated boulders on a cobble bed. At the same time, objective criteria are needed to describe the ability of varying bank materials to withstand erosive stresses. A quantitative explanation of the meandering process will require balancing the erosive stress produced by tangential shear of the flowing water and the comparable resisting stress provided by the bank material.

It is particularly important that critical field observations be tied to theoretical and laboratory studies. A few fundamental concepts can unify a vast amount of empirical observation if the observations include those critical measurements which make possible an adequate test of theory.

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APPENDIX. SHAPES OF MEANDER WAVES IN ALLUVIAL PLAINS

Appendix.—Shapes of Meander Waves in Alluvial Plains

(*Note:* Reaches of river chosen from maps to represent (a) a single meander length in which the S^3 curve was reasonably symmetrical; (b) the samples would include a large range in sizes of rivers. Each sample is considered representative of the local reach of river.)

River and location	Elevation (ft)	Range in width (ft)	Mean width (W) (ft)	Meander length (λ) (ft)	Arc distance (ft)	Mean radius (r_m) (ft)	Amplitude (A) (ft)	Sinuosity	Map quadrangle (scale = 1:24,000)
Sacramento R nr Chico, Calif.	110	250–550	550*	6700	8400	2050	2150	1.25	Ord Ferry, Calif.
Ninnescah R nr Belle Plain, Kans.	1200	80–160	260*	1350	1700	360	400	1.26	Belle Plain, Kans.
New R nr Brawley, Calif.	150	70–90	250*	1475	1750	210	400	1.18	Brawley, Calif.
Missouri R nr Buckner, Mo.	700	350–1550	1850*	14,600	24,300	3150	8850	1.66	Buckner, Mo.
Missouri R nr Buckner, Mo.	690	550–1950	1600*	37,100	41,800	15,550	7700	1.13	Buckner, Mo. Camden, Mo.
Kansas R nr Eudora	790	220–800	1000*	12,750	14,400	4300	2850	1.12	Lawrence E. Kans. Eudora, Kans.
Kissimmee R nr Okeechobee, Fla.	18	85–155	180*	1050	2900	205	1250	2.76	Okeechobee N.W., Fla.
Arkansas R nr Mulvane, Kans.	1190	220–360	500*	5650	7900	1350	2150	1.40	Mulvane, Kans.
Colorado R nr Blythe, Calif.	270	450–1650	1050*	25,300	33,300	7300	8900	1.32	Blythe N.E. Calif. Ariz.
San Joaquin R nr Patterson, Calif.	35	120–280	400*	2400	4550	610	1600	1.90	Brush Lake, Calif.
James R nr Forestburg, S. D.	1220	60–90	160*	2220	3600	450	1275	1.62	Forestburg, S. D.
James R nr Clayton, S. D.	1190	80–150	230*	1550	2600	265	900	1.68	Tschetter, S. D.
Souris R	1480	30–60	150*	1100	2500	160	720	2.27	Voltaire, N. D.
Missouri R nr Lexington, Mo.	690	450–1150	1350*	24,400	29,000	6000	7100	1.19	Camden, Mo. Lexington, Mo.
Sacramento R nr Glenn, Calif.	88+	250–450	950*	7600	10,800	2300	3200	1.42	Glenn, Calif. Llanos Seco, Calif.
Red R nr Campti, La.	100	400–1200	1400*	11,400	18,700	2130	7000	1.64	(scale 1:62,500) Campti, La.
Henrys Fork nr Menan, Idaho	4810	180–300	340*	2700	6100	850	2300	2.26	Menan Buttes, Idaho
Henrys Fork nr Menan, Idaho	4812	110–200	280*	1950	3300	410	1200	1.69	Menan Buttes, Idaho
Cedar R nr Belgrade, Neb.	1700	60–130	200*	3050	4550	750	1450	1.49	Belgrade, Neb.
Cedar R nr Belgrade, Neb.	1650	90–150	210*	2300	2900	575	820	1.26	Belgrade, Neb.
Mississippi R nr Wynnburg, Tenn.	265±	2132–5460	4940*	38,580	60,050	9620	27,400	1.55	*** Map 6, miles 92–102

Wind R nr Dubois, Wyo., Dumb Cowboy Reach		22–48	30	205	310	41	94	1.51	Plane-table map, authors
Squaw Cr nr Lander, Wyo.			8	82	115	17	40	1.40	Plane-table map, authors
Little Pipe Cr nr Westminster, Md.		10–18	12	145	192	35	58	1.32	Plane-table map, authors
Baldwin Cr nr Lander, Wyo.		10–18	14	185	245	45	72	1.32	Plane-table map, authors
Little Sandy nr Elkhorn, Wyo.		10–16	13	248	273	76	55	1.10	Plane-table map, authors
Buffalo Fork at Black Ranger Sta.		70–115	80	990	1230	308	310	1.24	Plane-table map, authors
Baldwin Cr nr Lander, Wyo.		12–24	15	165	205	43	48	1.24	Plane-table map Leopold and Miller
Mississippi R nr Smithland, La.	15	4900–2180	3330	65,000	75,400	15,700	19,850	1.16	*** Maps 38, 38, miles 770–785
Mississippi R nr Lake Providence, La.	85	2200–5800	3500	53,500	75,800	9900	11,400	1.42	*** Maps, 26, 27, miles 550–565
Mississippi R nr Rosedale, Miss.	145	4300–2440	3210	40,300	47,000	11,200	11,760	1.17	*** Maps, 19, 20, miles 387–396
New Fork nr Pinedale, Wyo. (Hailstone Reach)		48–76	62	745	960	163	225	1.29	Plane-table map, authors
Model, U. S. Waterways Expt. Sta.		2–4.8	3.3	34	43.5	8.9	13.0	1.28	Meandering of alluvial rivers, Friedkin, 1945 — Test 4, Plate 41
Model, U. S. Waterways Expt. Sta.		1.2–3.2	1.6	28.2	36.0	7.2	9.5	1.28	Test 1, Plate 29
Model, U. S. Waterways Expt. Sta.		2.3–3.0	2.6	29.5	34.2	7.0	8.5	1.16	Test 3, Plate 37
Moyen R nr Castets, France		627–1120	870	9040	12,200	2000	3760	1.36	Fargue, 1908
Coosa River	440	650–400	520	8976	18,500	1700	6700	2.06	Riverside, Ala.
Kansas River	780	850–170	510	16,400	26,100	3150	8550	1.59	Eudora, Kans.
Red River	100	500–1000	750	10,000	17,500	2000	5000	1.75	Campti, La.
Henrys Fork, nr Menan, Idaho	4810	350–150	250	2700	5500	650	2000	2.04	Menan Buttes, Idaho
Sacramento River	85	600–220	410	6100	11,000	1700	3700	1.80	Glenn and Llano Seco, Calif.
Kissimmee River	18	130–110	120	1100	2100	225	600	1.91	Okeechobee N.W., Fla.
Sacramento River	120	700–350	525	8340	11,500	1400	3600	1.38	Ord Ferry, Calif.
James River	1220		110	2220	2600	600	600	1.17	Forestburg, S. D.
Souris River, N. D.	1610		140	1800	3200	440	1040	1.78	** Sheet A, mile 12, 1926
Souris River	1590		140	1840	4000	480	1240	2.17	** Sheet A, mile 40, 1926
Souris River	1560		120	840	1600	220	500	1.90	** Sheet Cm mile 123, 1926
Souris River	1430		100	1140	2400	300	800	2.10	** Sheet K, mile 290, 1930
Red River, La.	40		600	7400	8800	1640	2100	1.19	*** Mile 33.0

* Estimated bankfull width

** Plan and profile of Souris (Mouse) River, International Boundary to Verendrye, N. D. (Advance sheets): Department of the Interior, Geological Survey, printed 1926

*** Mississippi River Commission, 1935, Maps of the Mississippi River, Cairo, Ill., to the Gulf of Mexico, La.; scale, 1:62,500: Vicksburg