Risk and Economic Analysis: Flood Frequency Estimation

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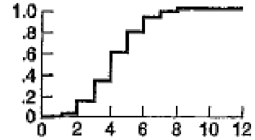
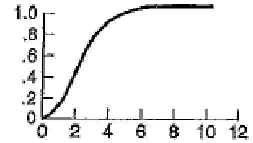
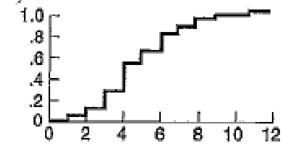
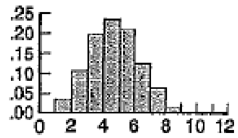
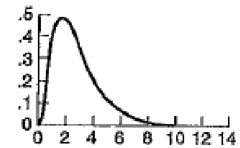
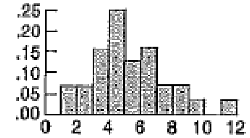
Water Resources Engineering

Concept	Population value, Discrete case	Population value, Continuous case	Sample value
Cumulative distribution function (cdf)	 Describes the probability that a random variable is less than or equal to a specified value x	 Describes the probability that a random variable is less than or equal to a specified value x	 Empirical distribution function (edf): describes the observed frequency of a random variable being less than or equal to a specified value x
Probability mass function (pmf) and probability density function (pdf)	 pmf: the probability that X is equal to k	 pdf: first derivative of the cumulative distribution function	 Histogram: observed frequency with which random variable X falls into the assigned ranges
Mean, average, or expected value	$\sqrt{0.368}$	$f(x) \equiv \frac{dF(x)}{dx}$ $\mu \equiv \int_{-\infty}^{\infty} x f(x) dx$	$\overline{X} \equiv \sum_{i=1}^{n} \frac{X_i}{n}$
Variance	$\sigma^2 \equiv \sum_{i=1}^{\infty} P(X = x_i)(x_i - \mu)^2$	$\sigma^2 \equiv \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$	$S^2 \equiv \sum_{i=1}^{n} \frac{(X_i - \overline{X})^2}{n-1}$
kth central moment	$M_k \equiv \sum_{i=1}^{\infty} P(X = x_i)(x_i - \mu)^k$	$M_k \equiv \int_{-\infty}^{\infty} (x-\mu)^k f(x)dx$	$\widetilde{M}_k \equiv \sum_{i=1}^{n} \frac{(X_i - \overline{X})^k}{n}$
Standard deviation	$\sigma \equiv \sqrt{\sigma^2}$		$S \equiv \sqrt{S^2}$
Coefficient of variation or relative standard deviation (if $\mu \neq 0$)	$CV \equiv \frac{\sigma}{\mu}$		$CV \equiv \frac{S}{\overline{X}}$
Coefficient of skew (a measure of asymmetry)	$\gamma \equiv \frac{M_3}{\sigma^3}$		$G \equiv \frac{\widetilde{M}_3}{S^3}$
Quantiles	x_p is any value of X that has the properties that $P[X < x_p] \leq p$ $P[X > x_p] \leq 1 - p$		$\hat{x}_p$ is the pth quantile of edf
Median (useful for describing central tendency regardless of skewness)	$x_{0.5}$ Any value of X that has the property that $P[X < x_p] \leq 0.5$ $P[X > x_p] \leq 0.5$		$\hat{X}_{0.5}$ The middle observation in a sorted sample, or the average of the two middle observations if the sample size is even.

Distribution	Probability density function	Range	Parameter-Moment relations
Normal	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$	$-\infty < x < \infty$	
Log-normal	$f(x) = \frac{1}{\sqrt{2\pi} x \sigma_{\ln x}} e^{-(\ln x - \mu_{\ln x})^2/(2\sigma_{\ln x}^2)}$	$x > 0$	$\mu_{\ln x} = \frac{1}{2}\ln\left[\frac{\mu_x^2}{1+\Omega_x^2}\right]$ $\sigma_{\ln x}^2 = \ln(1+\Omega_x^2)$ $\Omega_x = \sigma_x/\mu_x$
Exponential	$f(x) = \lambda e^{-\lambda x}$	$x \geq 0$	$\lambda = \frac{1}{\mu_x}$
Gamma	$f(x) = \frac{\lambda^\beta x^{\beta-1} e^{-\lambda x}}{\Gamma(\beta)}$ where Γ = gamma function	$x \geq 0$	$\lambda = \frac{\mu_x}{\sigma_x^2}$, $\beta = \frac{\mu_x^2}{\sigma_x^2} = \frac{1}{C_v^2}$
Extreme Value Type I	$f(x) = \frac{1}{\alpha} e^{-(x-\beta)/\alpha - e^{-(x-\beta)/\alpha}}$	$-\infty < x < \infty$	$\alpha = \sqrt{6}\sigma_x/\pi$ $\beta = \mu_x - 0.5772\alpha$
Log Pearson Type III	$f(x) = \frac{\lambda^\beta (y-\epsilon)^{\beta-1} e^{-\lambda(y-\epsilon)}}{x\Gamma(\beta)}$ where $y = \log x$	$\log x \geq \epsilon$	$\lambda = \frac{s_y}{\sqrt{\beta}}$, $\beta = \left[\frac{2}{G_s(y)}\right]^2$ $\epsilon = \bar{y} - s_y\sqrt{\beta}$ (assuming $G_s(y)$ is positive)

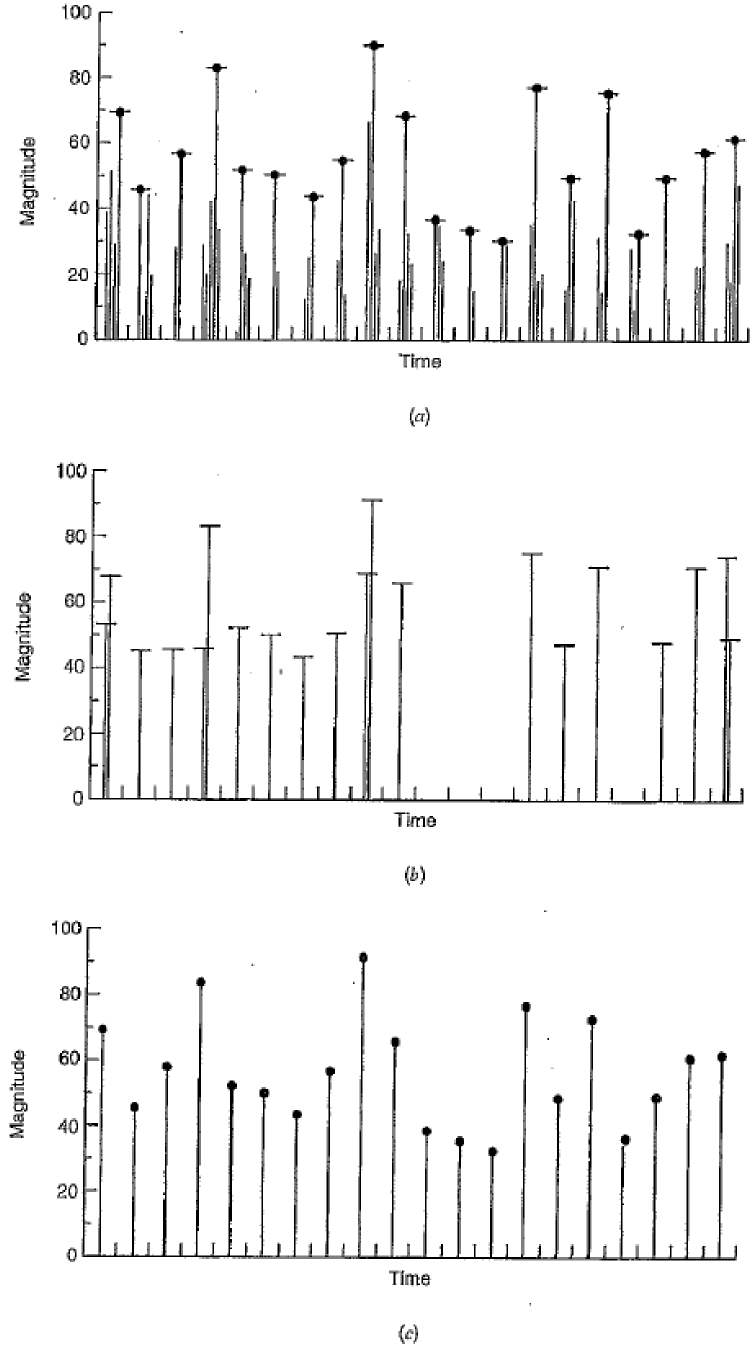


Figure 10.3.3 Hydrologic data arranged by time of occurrence. (*a*) Original data; $N = 20$ years; (*b*) Annual exceedances; (*c*) Annual maxima (from Chow (1964)).

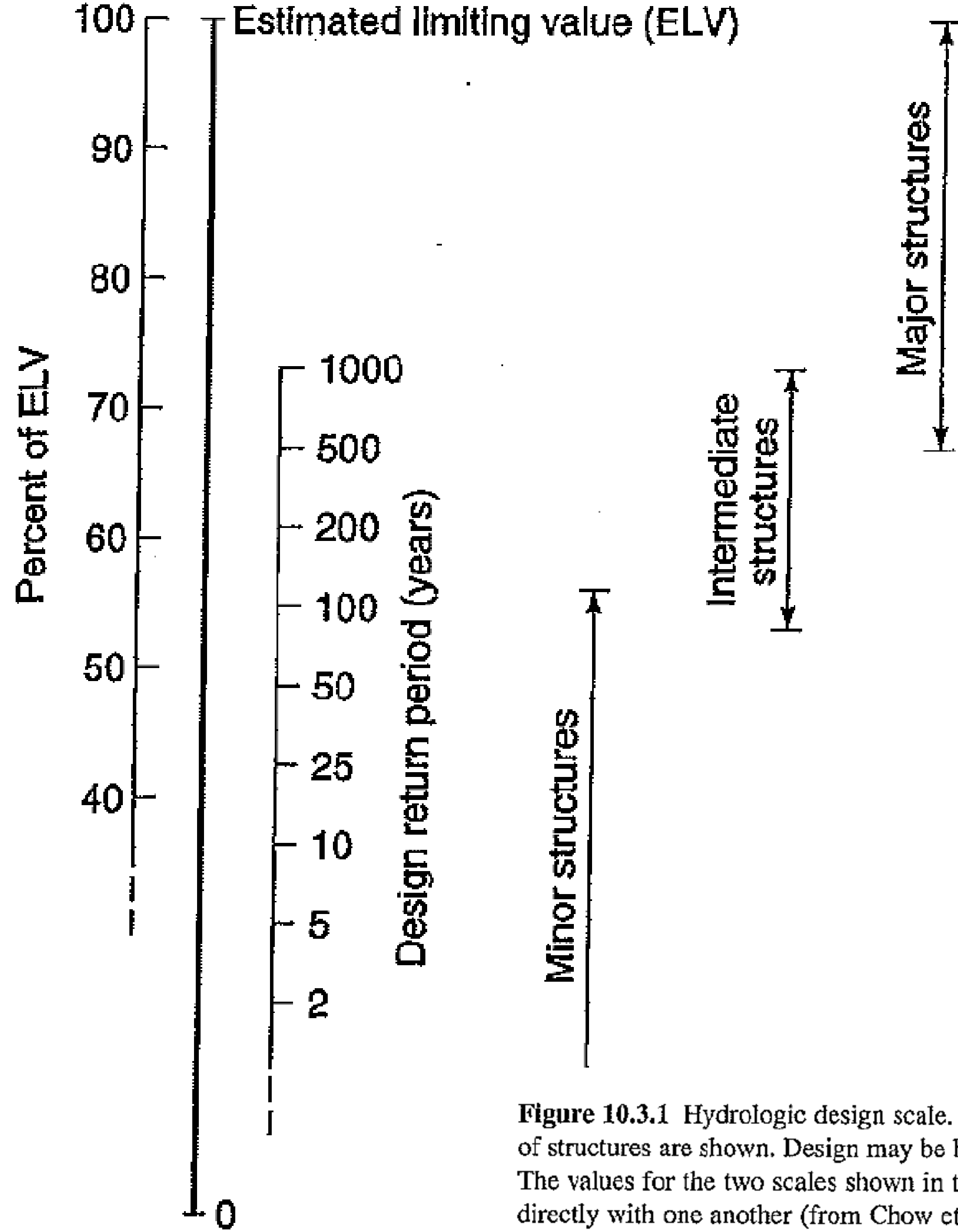


Figure 10.3.1 Hydrologic design scale. Approximate ranges of the design level for different types of structures are shown. Design may be based on a percentage of the ELV or on a design return period. The values for the two scales shown in the diagram are illustrative only and do not correspond directly with one another (from Chow et al. (1988)).

Table 10.3.1 Generalized Design Criteria for Water-Control Structures

Type of structure	Return period (Years)	ELV (%)
Highway culverts		
Low traffic	5–10	—
Intermediate traffic	10–25	—
High traffic	50–100	—
Highway bridges		
Secondary system	10–50	—
Primary system	50–100	—
Farm drainage		
Culverts	5–50	—
Ditches	5–50	—
Urban drainage		
Storm sewers in small cities	2–25	—
Storm sewers in large cities	25–50	—
Airfields		
Low traffic	5–10	—
Intermediate traffic	10–25	—
High traffic	50–100	—
Levees		
On farms	2–50	—
Around cities	50–200	—
Dams with no likelihood of loss of life (low hazard)		
Small dams	50–100	—
Intermediate dams	100+	—
Large dams	—	50–100
Dams with probable loss of life (significant hazard)		
Small dams	100+	50
Intermediate dams	—	50–100
Large dams	—	100
Dams with high likelihood of considerable loss of life (high hazard)		
Small dams	—	50–100
Intermediate dams	—	100
Large dams	—	100

Source: Chow et al. (1988).

IOWA

The Great Iowa Flood of 2008

Iowa City

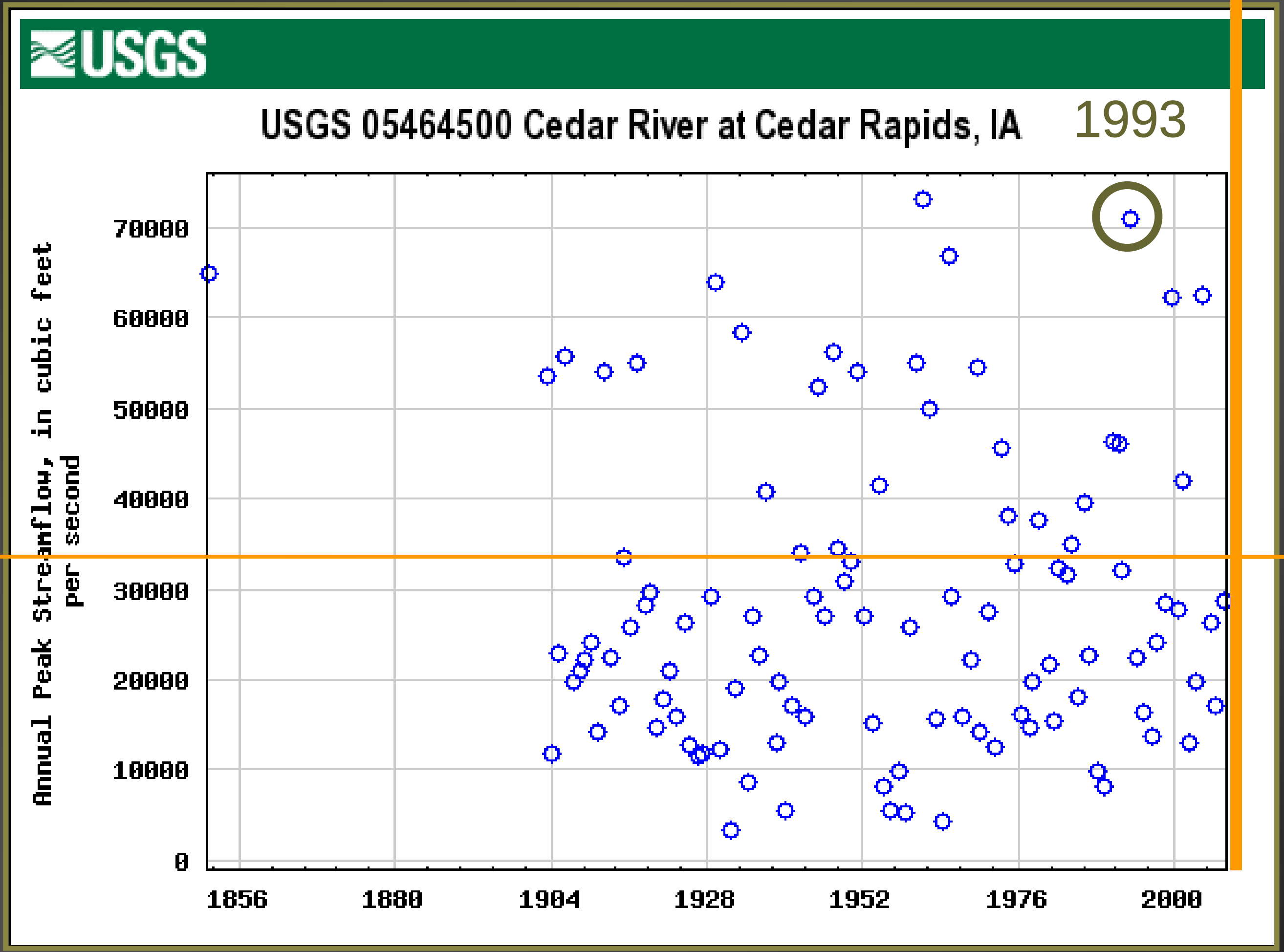
Cedar Rapids

Flood Frequency

How rare (or common) are floods of this magnitude?

Annual Flood Time Series

2008
145,000 cfs



31 floods
in 106 years

Flood

~33,600 cfs

No flood

Probability of a Flood?

- Sample estimate

$$m = 31 \text{ (floods) in } N = 106 \text{ (years)}$$

  - Estimate relative frequency of the event

$$p = \frac{m}{N+1} = \frac{31}{107} = 0.290$$

~29% chance of a flood each year

Time Interval Between Floods?

- Average recurrence interval

$$T = \frac{1}{p} = \frac{1}{0.290} = 3.5 \text{ (years)}$$

Average time between floods is ~3.5 years

Assumes floods are random occurrences

Flood Frequency Analysis

- Hypothesize the mathematical form of the relations between Q and p
- Estimate exceedance probability p_i for each peak discharge Q_i in the sample
- Plot Q_i versus p_i
  - Use specialized graph paper, or plot transformed variables on rectilinear paper
- Fit the mathematical model

Flood Probability Model

- Form (Extreme Value Type I)

$$Q(p) = \alpha - \beta \ln(-\ln(1 - p))$$

  - $Q(p)$ flood magnitude with probability p
  - p exceedance probability
  - α, β model parameters

Flood Probability Model

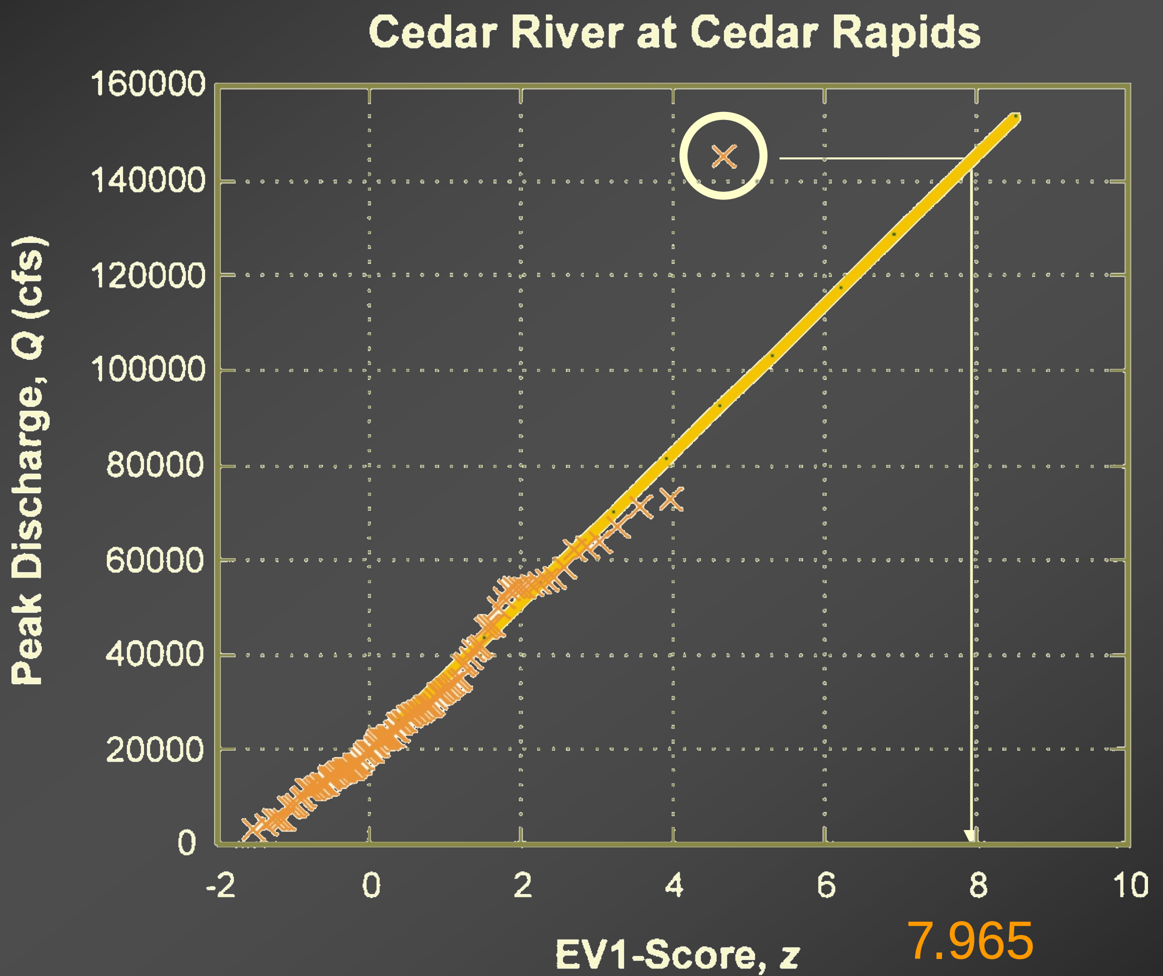
- Define the EV1 score:

$$z = -\ln(-\ln(1-p))$$

- Linearized flood probability model:

$$Q(p) = \alpha + \beta z$$

Flood Frequency Analysis



Fitted EV1 Flood Probability Model

2008 Cedar Rapids Flood

- Flood probability

$$7.965 = -\ln(-\ln(1-p))$$

$$p = 1 - e^{-e^{-7.965}} = 0.0003475$$

- Average recurrence interval:

$$T = \frac{1}{p} = \frac{1}{0.0003475} = 2900 \text{ years!}$$

Cedar River at Cedar Rapids

- Flood frequency estimates

T (years)	***p*** ()	***Q(p)*** (cfs)
25	0.04	70,000
50	0.02	81,000
100	0.01	92,000
200	0.005	103,000
500	0.002	118,000
1000	0.001	128,000
5000	0.0002	154,000

Summary

- Engineers use mathematical models to estimate flood probabilities (risk) and average recurrence intervals from a flood data sample
- The 2008 flood at Cedar Rapids was an extraordinary rare event
  - ~3000 year recurrence interval
- In contrast, at Cedar Rapids, 1993 Mississippi River flood was a much more common event
  - ~30 year recurrence interval