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Homework 1

Due: January 27, 2005
(Exam on February 17, 2005)

Read Appendix A: Vector and Tensors

1. If $\vec{v} = (z-y)\vec{e}_x + (x-z)\vec{e}_y - (y-x)\vec{e}_z$ compute $\nabla \cdot \vec{v}$ and $\nabla \times \vec{v}$.

2. Verify in RCCS that

   a) $\nabla \cdot (\vec{u} + \vec{v}) = \nabla \cdot \vec{u} + \nabla \cdot \vec{v}$

   b) $\nabla \cdot (\nabla \times \vec{v}) = 0$

   c) $\nabla \times (\nabla s) = \vec{0}$

   d) $\nabla \cdot \left(s\vec{\vec{D}} \right) = s\left(\nabla \cdot \vec{\vec{D}} \right) + (\nabla s) \cdot \vec{\vec{D}}$

   e) $\frac{1}{2} \nabla(\vec{v} \cdot \vec{v}) = \vec{v} \cdot \nabla\vec{v} + \vec{v} \times (\nabla \times \vec{v})$

   f) $\nabla \cdot (\vec{u}\vec{v}) = (\nabla \cdot \vec{u})\vec{v} + \vec{u} \cdot \nabla\vec{v}$

3. Use the generalized definition of gradient to obtain an expression for $\nabla\vec{v}$ in cylindrical coordinates.

4. From the definitions of divergence, gradient and identity tensor, $\vec{\vec{I}}$, show that $\nabla \cdot (\nabla\vec{v})^{+} = \nabla(\nabla \cdot \vec{v})$ for any vector field $\vec{v}$.

5. Show that the following vector field is irrotational and determine a scalar potential $a(x, y, z)$ such that $\vec{v} = \nabla a$, $\vec{v} = 2x^2\vec{i} - 2yz\vec{j} - (y^2 + 3)\vec{k}$.