\[ M_1 \times 334.86 \text{kJ} - 100 \times 104.88 \text{kJ} = \left[ M_2 - 100 \right] \times 3069.3 \text{kJ} \]

Thus

\[ M_2 (3069.3 - 334.86) = 100 \times (3069.3 - 104.88) \]

\[ M_2 = 108.41 \text{ kg}, \text{ and } \Delta M = M_2 - M_1 = 8.41 \text{ kg} \text{ of steam added.} \]

2.7 (a) Consider a change from a given state 1 to a given state 2 in a closed system. Since initial and final states are fixed, \( U_1, U_2, V_1, V_2, P_1, P_2, \) etc. are all fixed. The energy balance for the closed system is

\[ U_2 - U_1 = Q + W - \int PdV = Q + W \]

where \( W = W_0 - \int PdV \) = total work. Also, \( Q = 0 \) since the change of state is adiabatic. Thus, \( U_2 - U_1 = W \).

Since \( U_1 \) and \( U_2 \) are fixed (that is, the end states are fixed regardless of the path), it follows that \( W \) is the same for all adiabatic paths. This is not in contradiction with Illustration 2.5-6, which established that the sum \( Q + W \) is the same for all paths. If we consider only the subset of paths for which \( Q = 0 \), it follows, from that illustration that \( W \) must be path independent.

(b) Consider two different adiabatic paths between the given initial and final states, and let \( W^* \) and \( W^{**} \) be the work obtained along each of these paths, i.e.,

Path 1: \( U_2 - U_1 = W^* \); Path 2: \( U_2 - U_1 = W^{**} \)

Now suppose a cycle is constructed in which path 1 is followed from the initial to the final state, and path 2, in reverse, from the final state (state 2) back to state 1. The energy balance for this cycle is

\[ U_2 - U_1 = W^* \]

\[ -(U_2 - U_1) = -W^{**} \]

\[ 0 = W^* - W^{**} \]

Thus if the work along the two paths is different, i.e., \( W^* \neq W^{**} \), we have created energy!

2.8 System = contents of tank at any time

mass balance: \( M_2 - M_1 = \Delta M \)

energy balance: \( (\dot{M}\dot{U})_2 - (\dot{M}\dot{U})_1 = \Delta M\dot{H}_{in} \)
(a) Tank is initially evacuated ⇒ \( M_1 = 0 \)

Thus \( M_2 = \Delta M \), and

\[
\dot{U}_2 = \dot{H}_2 = \dot{H}(5 \text{ bar, } 370^\circ \text{C}) = 3209.6 \text{ kJ/kg} \quad \text{(by interpolation)}.
\]

Then \( \dot{U}_2 = \dot{U}(P = 5 \text{ bar, } T = ?) = 3209.6 \text{ kJ/kg} \). By interpolation, using the Steam Tables (Appendix III) \( T = 548^\circ \text{C} \)

\[
\dot{V}(P = 5 \text{ bar, } T = 548^\circ \text{C}) \approx 0.756 \text{ m}^3/\text{kg}
\]

Therefore \( M = \dot{V}/\dot{V} = 1 \text{ m}^3/(0.756 \text{ m}^3/\text{kg}) = 1.3228 \text{ kg} \).

(b) Tank is initially filled with steam at 1 bar and 150°C

⇒ \( \dot{V}_1 = \dot{V}(P = 1 \text{ bar, } T = 150^\circ \text{C}) = 1.94 \text{ m}^3/\text{kg} \) and

\( \dot{U}_1 = 2583 \text{ kJ/kg} \),

\( M_1 = \dot{V}/\dot{V} = 1/\dot{V} = 0.5155 \text{ kg} \). Thus, \( M_2 = 0.5155 + \Delta M \text{ kg} \). Energy balance is

\[
M_2 \dot{U}_2 - 0.5155 \times 2583 = (M - 0.5155) \times 3209.6
\]

Solve by guessing value of \( T_2 \), using \( T_2 \) and \( P_2 = 5 \text{ bar} \) to find \( \dot{V}_2 \) and \( \dot{U}_2 \) in the Steam Tables (Appendix III). See if energy balance and \( M_2 = 1 \text{ m}^3/\dot{V}_2 \) are satisfied. By trial and error: \( T_2 \approx 425^\circ \text{C} \) and \( M_2 \approx 1.563 \text{ kg} \) of which 1.323 kg was present in tank initially. Thus, \( \Delta M = M_2 - M_1 = 0.24 \text{ kg} \).

2.9 a) Use kinetic energy = \( mv^2/2 \) to find velocity.

\[
1 \text{ kg} \times \frac{v^2}{2} \text{ m}^2/\text{sec}^2 = 1000 \text{ J} = 1000 \text{ kg} \text{ m}^2/\text{sec}^2
\]

so \( v = 44.72 \text{ m/sec} \)

b) Heat supplied = specific heat capacity \times temperature change, so

\[
1000 \text{ g} \times \frac{1 \text{ mol}}{55.85 \text{ g}} \times 25.10 \text{ J/mol} \cdot \text{K} \times \Delta T = 1000 \text{ J} \quad \text{so } \Delta T = 2.225 \text{ K.}
\]

2.10 System = resistor

Energy balance: \( dU/dt = \dot{W}_s + \dot{Q} \)

where \( \dot{W}_s = E \cdot I \), and since we are interested only in steady state \( dU/dt = 0 \).

Thus

\[
-\dot{Q} = \dot{W}_s = 1 \text{ amp} \times 10 \text{ volts} = 0.2 \times (T - 25^\circ \text{C}) \text{ J/s}
\]

and 1 watt = 1 volt \times 1 amp = 1 J/s.

\[
\Rightarrow T = \frac{10 \text{ watt} \times 1 \text{ J/s} \cdot \text{watt}}{0.2 \text{ J/s} \cdot \text{K}} + 25^\circ \text{C} = 750^\circ \text{C}
\]

2.11 System = gas contained in piston and cylinder (closed)

Energy balance: \( U|_{V_2} - U|_{V_1} = Q + \int PdV \)

(a) \( V \) = constant, \( \int PdV = 0 \), \( Q = U|_{V_2} - U|_{V_1} = N(U|_{V_2} - U|_{V_1}) = NC_v(T_2 - T_1) \)

From ideal gas law

\[
N = \frac{PV}{RT} = \frac{114,367 \text{ Pa} \times 0.120 \text{ m}^3}{8.314 \text{ Pa} \cdot \text{m}^3/\text{mol} \cdot \text{K} \times 298 \text{ K}} = 5539 \text{ mol} \quad \text{(see note following)}
\]
M.B. \[ M^f - M^i = \Delta M_i \]
\[ M^i = M^i_L + M^i_v ; \quad M^i_L = \frac{200 \text{ liters}}{V^i_L} = 194.932 \text{ kg}; \]
\[ M^i_v = \frac{60 \text{ m}^3 - 200 \text{ liters}}{V^i_v} = 14.476 \text{ kg and so } M^f = 209.408 \text{ kg} \]

E.B.
\[ M^f \dot{U}^f - M^i \dot{U}^i = \Delta M \dot{H}_k \]
\[ (M^i_L \dot{U}^i_L + M^i_v \dot{U}^i_v) = \left[ M^f_L \dot{U}^f_L + M^f_v \dot{U}^f_v \right] = [M^f_L + M^f_v] [0.1 \dot{H}_{L,\text{in}} + 0.9 \dot{H}_{v,\text{in}}] \]

Total internal energy of steam + water in the tank
194.932×313.0 + 14.476×2475.9 = 9.686×10^4 kJ

Properties of steam entering, 90% quality
Specific volume = \( \dot{V}_{\text{in}} = 0.1 \times 1.061 \times 10^{-3} + 0.9 \times 0.8857 = 0.797 \text{ m}^3/\text{kg} \)
Specific enthalpy = \( \dot{H}_{\text{in}} = 0.1 \times 504.70 + 0.9 \times 2706.7 = 2.486 \times 10^3 \text{ kJ/kg} \)

Also have that \( V = 60 \text{ m}^3 = M^i_L \dot{V}^i_L + M^i_L \dot{V}^i_L \).

This gives two equations, and two unknowns, \( M^i_L \) and \( M^i_v \).

The solution (using MATHCAD) is \( M^i_L = 215.306 \text{ kg} \) and \( M^i_v = 67.485 \text{ kg} \).

Therefore, the fraction of the tank contents that is liquid by weight is 0.761.

2.15 System = contents of both chambers (closed, adiabatic system of constant volume. Also \( W = 0 \)).

Energy balance: \( U(t_2) - U(t_1) = 0 \) or \( U(t_2) = U(t_1) \)
(a) For the ideal gas \( u \) is a function of temperature only. Thus, \( U(t_2) = U(t_1) \Rightarrow T(t_2) = T(t_1) = 500 \text{ K} \). From ideal gas law
\[ P_1 V_1 = N_1 RT_1 \quad \text{but} \quad N_1 = N_2 \text{ since system is closed} \]
\[ P_2 V_2 = N_2 RT_2 \quad \text{and} \quad T_1 = T_2 \text{ see above} \]
\[ V_2 = 2V_1 \text{ see problem statement.} \]
\[ \Rightarrow P_2 = \frac{1}{2} P_1 = 5 \text{ bar} = 0.5 \text{ MPa} \Rightarrow T_2 = 500 \text{ K}, P_2 = 0.5 \text{ MPa} \]

(b) For steam the analysis above leads to \( U(t_2) = U(t_1) \) or, since the system is closed \( \dot{U}(t_2) = \dot{U}(t_1) \), \( \dot{V}(t_2) = 2\dot{V}(t_1) \). From the Steam Tables, Appendix III,
\[ \dot{U}(t_1) = \dot{U}(T = 500 \text{ K}, P = 1 \text{ MPa}) = \dot{U}(T = 2268.5^\circ \text{C}, P = 1 \text{ MPa}) \approx 2669.4 \text{ kJ/kg} \]
\[ \dot{V}(t_1) = \dot{U}(T = 2268.5^\circ \text{C}, P = 1 \text{ MPa}) \approx 0.2204 \text{ m}^3/\text{kg} \]

Therefore \( \dot{U}(t_2) = \dot{U}(t_1) = 2669.4 \text{ kg/kg} \) and \( \dot{V}(t_2) = 2\dot{V}(t_1) = 0.4408 \text{ m}^3/\text{kg} \). By, essentially, trial and error, find that \( T \sim 216.3^\circ \text{C} \), \( P \sim 0.5 \text{ MPa} \).
(c) Here \( U(t_2) = U(t_1) \), as before, except that \( U(t_1) = U^I(t_1) + U^{II}(t_1) \), where superscript denotes chamber.

Also, \( M(t) = M^I(t_1) + M^{II}(t_1) \) \{mass balance\} and

\[
\dot{V}(t_2) = 2V_t / M(t_2) = 2V_t / [M^I(t_1) + M^{II}(t_1)]
\]

For the ideal gas, using mass balance, we have

\[
\frac{P_2(2V_1)}{T_2} = \frac{P^I_1V_1}{T_1} + \frac{P^{II}V_1}{T_1^{II}} \Rightarrow \frac{2P_2}{T_2} = \frac{P^I_1}{T_1} + \frac{P^{II}}{T_1^{II}} \tag{1}
\]

Energy balance: \( N_t U_2 = N^I_1 U^I_1 + N^{II} U^{II}_1 \)

Substitute \( \dot{U} = U_0 + N C_v (T - T_0) \), and cancel terms, use \( N = PV/RT \) and get

\[ 2P_2 = P^I_1 + P^{II} \tag{2} \]

Using Eqns. (1) and (2) get \( P_2 = 7.5 \times 10^5 \) Pa = 0.75 MPa and \( T_2 = 529.4 \) K (256.25° C).

(d) For steam, solution is similar to (b). Use Steam Table to get \( M^I_1 \) and \( M^{II}_1 \) in terms of \( V \).

Chamber 1: \( \dot{U}^I_1 = 2669.4 \) kJ/kg \( ; \dot{V}^I_1 = 0.2204 \) m³/kg \( ; \)

\( M^I = V_1 / \dot{V}^I_1 = 4537 V_1 \)

Chamber 2: \( \dot{U}^{II}_1 = \dot{U}(T = 600 \) K, \( P = 0.5 \) MPa) = 28459 kJ/kg \( ; \)

\( \dot{V}^{II}_1 = 0.5483 \) m³/kg \( ; M^{II} = 1.824V_1 = V_1 / \dot{V}^{II}_1 \)

Thus, \( \dot{V}_2 = \frac{2V_1}{M^I + M^{II}} = \frac{2V_1}{4537V_1 + 1.824V_1} = 0.3144 \) m³/kg \( ; \)

\( \dot{U}_2 = \left( M^I \dot{U}^I_1 + M^{II} \dot{U}^{II}_1 \right) / \left( M^I + M^{II} \right) = 2720.0 \) kJ/kg

By trial and error: \( T_2 \approx 302^\circ \) C (575 K) and \( P \approx 0.76 \) MPa.

2.16 System: contents of the turbine (open, steady state)

(a) adiabatic

mass balance: \( \frac{dM}{dt} = 0 = \dot{M}_1 + \dot{M}_2 \Rightarrow \dot{M}_2 = -\dot{M}_1 \)

energy balance: \( \frac{dU}{dt} = 0 = \dot{M}_1 \dot{H}_1 + \dot{M}_2 \dot{H}_2 + \oint 0 + \dot{W}_s - P \oint 0 \)

\( \Rightarrow \dot{W}_s = -\dot{M}_1 (\dot{H}_1 - \dot{H}_2) = -\dot{M}_1 (3450.9 - 2865.6) \) kJ/kg

\( = -\dot{M}_1 (5853 \times 10^5) \) J/kg

But \( \dot{W}_s = -7.5 \times 10^5 \) watt \( = -7.5 \times 10^5 \) J/s

\( \dot{M}_1 = \frac{-7.5 \times 10^5}{5853 \times 10^5} \) J/s \( = 1.281 \) kg/s \( = 4.613 \times 10^3 \) kg/h

(b) Energy balance is

\( \frac{dU}{dt} = 0 = \dot{M}_1 \dot{H}_1 + \dot{M}_2 \dot{H}_2 + Q + \dot{W}_s - P \oint 0 \)

\[ T_2 = T_1 - \frac{5209 \text{ J/kmol}}{368 \text{ J/mol} \times 1000 \text{ mol/kmol}} = T_1 - 0.14^\circ \text{C} \]

Thus the kinetic energy term makes such a small contribution, we can safely ignore it.

(b) Mass balance on compressor (steady-state) \( 0 = \dot{N}_1 + \dot{N}_2 \)

\[
\begin{array}{c}
2.0 \times 10^6 \text{ Pa} \\
T_1 = 25^\circ \text{C}
\end{array} \xrightarrow{\text{compressor}} \begin{array}{c}
3.0 \times 10^6 \text{ Pa} \\
T_2 = ?
\end{array}
\]

Energy balance on compressor, which is in steady-state operation

\[ 0 = \dot{N}_1 H_1 + \dot{N}_2 H_2 + \dot{Q} + \dot{W} \Rightarrow \dot{W} = \dot{N}_1 C_p (T_2 - T_1) \]

adiabatic compressor

Can compute \( \dot{W} \) if \( T_2 \) is known or vice versa. However, can not compute both without further information.

\[
\begin{array}{c}
2.0 \times 10^6 \text{ Pa} \\
T_2 = ?
\end{array} \xrightarrow{\text{Gas cooler}} \begin{array}{c}
3.0 \times 10^6 \text{ Pa} \\
T_1 = 25^\circ \text{C}
\end{array}
\]

Analysis as above except that \( \dot{Q} \neq 0 \) but \( \dot{W} = 0 \).

Here we get

\[
\begin{align*}
0 &= \dot{N}_2 + \dot{N}_3 \\
\dot{Q} &= \dot{N}_1 C_p (T_1 - T_2) \\
\end{align*}
\]

Can not compute \( \dot{Q} \) until \( T_2 \) is known.

See solution to Problem 3.10.

2.32 a) Define the system to be the nitrogen gas. Since a Joule-Thomson expansion is isenthalpic, \( \dot{H}(T_1, P_1) = \dot{H}(T_2, P_2) \). Using the pressure enthalpy diagram for nitrogen, Figure 2.4-3, we have

\[ \dot{H}(135 \text{ K}, 20 \text{ MPa}) = 153 \text{ kJ/kg and then } T_2 = T(P_2 = 0.4 \text{ MPa}, \dot{H} = 153 \text{ kJ/kg}) \]

From which we find that \( T = 90 \text{ K} \), with approximately 55\% of the nitrogen as vapor, and 45\% as liquid.

b) Assuming nitrogen to be an ideal gas (poor assumption), then the enthalpy depends only on temperature. Since a Joule-Thomson expansion is isenthalpic, this implies that the temperature is unchanged, so that the final state will be all vapor.

2.33 Plant produces \( 1.36 \times 10^9 \) kwh of energy per year

\( \Rightarrow \) Plant uses \( 1.36 \times 10^9 \times 4 = 5.44 \times 10^9 \) kwh of heat
1 kwh = 3.6 × 10^6 J

⇒ Plant uses \(3.6 \times 10^6 \frac{J/\text{year}}{\text{kwh}} \times 5.44 \times 10^9\) kwh = \(19.584 \times 10^{15}\) J/year

\[\Delta H \text{ of rock (total)} = M \cdot \dot{C}_p(T_f - T_i)\]

\[= 10^{12} \text{ kg} \times 1 \text{ J/g K} \times 1000 \text{ g/kg} \times (110 - 600) \text{ K}\]

\[= -490 \times 10^{15}\) J

⇒ \(19.58 \times 10^{15}\) J/year \(\times x\) years = \(490 \times 10^{15}\) J

\(x = 25.02\) years