Matrix Methods Summary
## Direct and Iterative Methods

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<td>Multigrid</td>
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### Ill-Conditioned Matrices

- Singular-Value Decomposition
- Numerous specialized methods for inverse problems
Matrix solution takes too long

- Modify numerical method for solution of PDE to make matrix have a simple form (e.g., banded, triangular, symmetric)
- Use SOR iteration acceleration
- Use fast iterative method
Matrix is too large to store

- Use grid with fewer points
- Modify numerical method to have banded matrix or matrix in only one direction (e.g., ADI)
- Use iterative method that doesn’t require storage of full matrix
- Use multigrid approach on parallel network
Iterative method doesn’t converge or converges slowly

- Check matrix condition number and diagonal dominance
- Use under-relaxation
- Use preconditioning to improve condition number and/or make matrix more diagonally dominant

\[(P^{-1}A)x = P^{-1}b\] where \(P\) is “close” to \(A\)
Matrix is ill-conditioned

- You made a mistake – go back and check
- Use double precision
- Use specialized method for inverse problem or SVD