Product Evaluation for Performance and the Effects of Variation

- to compare the performance of the product to the engineering specifications (or targets) developed earlier in the design project

- The process of Product Evaluation
  - Monitoring functional change
  - Goals of performance evaluation
  - Accuracy, variation, and noise
  - Modeling for performance evaluation
  - Tolerance analysis
  - Sensitivity analysis
  - Robust design
  - Design for cost (DFC)
  - Value Engineering
  - Design for manufacture (DFM)
  - Design for assembly (DFA)
  - Design for reliability (DFR)
  - Design for test and maintenance
  - Design for the environment
Benefits in refining the function model as the form is evolving
- The functions that the product must accomplish can be kept very clear by updating the functional breakdown.
  - Nearly every decision about the form of an object adds something, either desirable or undesirable to the function of the object.
- Tracking the evolution of function means continuously updating the flow models of energy, information, and materials.
  - These flows determine the performance of the product.
To evaluate the product design relative to targets set previously.

Factors must be supported by the evaluation of product performance:
- Evaluation must result in numerical measures of the product for comparison with the engineering requirement targets developed during the problem understanding.
  - Measurements must be of sufficient accuracy and precision for valid comparison.
- Evaluation should give some indication of which features of the product design to modify; and by how much in order to bring the performance on target.
- Evaluation procedures must include the influence of variations due to manufacturing, aging, and environmental changes.
  - Insensitivity to these “noises” while meeting the targets results in a robust, quality product.

Additional concepts for better design:
- Optimization; trade studies, accuracy, tolerances, sensitivity analysis and robust design.
How to Evaluate Performance?

- Performance can be evaluated using a model.
  - Graphical model for form evaluation
    - Sketches and layout drawings
  - Analytical model for function evaluation
    - Back-of-the-envelope analysis, engineering science analysis, or detailed computer simulation (optimization, FEM, etc.)
  - Physical model for function and form evaluation
    - Prototypes for proof-of-concept, proof-of-product and proof-of-production

- Example: Design of a Tank to hold liquid:
  - A customer’s requirement is to design the “best” tank to hold “exactly” 4 m$^3$ of liquid.
  - Assume that conceptual design of the tank resulted in a cylindrical shape with an internal radius $r$ and an internal length $l$.
  - Analytical model: $V = \pi r^2 l$
    - $r^2l = 1.27$ m$^3$

*More thoughts required:
1. Other quality measures that may limit the potential $r$ and $l$ values.
   - weight, size targets, manufacturability, environment, etc.
2. What is meant by the terms “best” and “exactly”?
   - accuracy, variation, and noise
Accuracy, Variation, and Noise

- The purpose of modeling is to find the easiest method by which to evaluate the product for comparison with the engineering targets using available resources.

- Two types of errors in any model
  - Errors due to inaccuracy
  - Errors due to variation

- Accuracy
  - The correctness or truth of the model’s estimate
  - In case of distributed results, the best estimate (mean) will be a good predictor of product performance.
  - The variation in the results obtained from the model refers to statistical variation of the results about the mean value.
    - Precision, resolution, range and deviation are also used to refer to the distribution of the evaluation.
  - The obvious goal in modeling is to develop an accurate model with a small variation.
  - Accuracy tells “how much” whereas distribution tells “how sure”.

- Why concern the variation?
  - Each parameter that defines the product or process has variation and so each may vary greatly from the desired mean.
Examples of Variation

- Remember that, during production, not all samples of the product:
  - are exactly the same size;
  - are made of exactly same material; or
  - behave in exactly the same way.

- We have to consider how such variations affect the performance in the design process.
  - Deterministic analytic models
  - Non-deterministic (or stochastic) analytical models that account for both the mean and the variation by using methods from probability and statistics
Effect of Variation on Product Quality

- A product is considered to be of high quality if its quality measures stay on target regardless of parameter variation due to manufacturing, aging, or environment.

- Control parameters vs. Noise as a source of variation
  - Control parameters:
    - parameters controllable by the designer, such as working environment, geometry, etc.
  - Noise:
    - Uncontrollable parameters
    - Noises affecting the design parameters
      - Manufacturing, or unit-to-unit variations
      - Aging, or deterioration, effects, including etching, corrosion, wear and other surface effects
      - Environmental, or external, conditions including all effects of the operating environment.
How to Deal With Noises

- Noises that affect the strength are often accounted for by using a “safety factor (or factor of safety).”
  - \[ FS = \frac{S_{al}}{\sigma_{ap}} \] (\( S_{al} \) = allowable strength; \( \sigma_{ap} \) = applied stress)
  - Rule-of-Thumb Factor of Safety (see appendix C)

- Keep noises small by tightening manufacturing variations (generally expensive)
- Add active controls that compensate for the variations (generally complex and expensive).
- Shield the product from aging and environmental effects (sometimes difficult and may be impossible).
- Make the product insensitive to the noises (robust design).
  - Key Philosophy of Robust Design:
    • Determine values for the parameters based on easy-to-manufacture tolerances and default protection from aging and environmental effects so that the best performance is achieved. The term, best performance, implies that the engineering targets are met and the product is insensitive to noise. If noise-insensitivity cannot be met by adjusting the parameters, then tolerances must be tightened or the product shielded from the effects of aging and environment.
Modeling for Performance Evaluation

- **Steps to give order to the considerations taken into account during evaluation:**
  1. Identify the output responses (i.e., critical or quality parameters) that need to be measured.
  2. Note how accurate the output needs to be.
  3. Identify the input signal, the control parameters and their limits, and noises.
  4. Understand analytical modeling capabilities.
  5. Understand the physical modeling capabilities
  6. Select the most appropriate modeling method.
  7. Perform the analysis or experiments.
  8. Verify the results.
Tolerance Analysis

- Theoretically, tolerance is assumed to represent ±3% standard deviations about the mean value, implying that 99.68% of all the samples should fall within the tolerance.

- Focus of tolerance design is the concern about tolerances on dimensions and other variables (i.e., material properties) that affect the product.
  - It is shown that only a fraction of the tolerances on a typical component actually affect its function.
Effect of Tighter Tolerances on the Manufacturing Cost

*Specification of tighter tolerances will increase the manufacturing cost.
- use nominal tolerance whenever possible.

Meaning of the Tolerances Specified on the Drawings:
1. It communicates information to manufacturing that is essential in helping to determine the manufacturing processes that will be used.
2. Tolerance information is used to establish quality-control guide-line. (conformance quality)
Additive Tolerance Stack-up

- Most common form of tolerance analysis.

*Example of Air shock-swingarm:*
When the joint is assembled,

\[ l_g = l_s - (l_b + 2 \times l_w) \]

- \( l_g \) = gap length
- \( l_s \) = distance between fingers
- \( l_b \) = bushing length
- \( l_w \) = washer thickness

Worst-case Analysis:
If \( l_b = 19.97 \text{ (min)} \), \( l_w = 1.95 \text{ (min)} \), \( l_s = 24.1 \text{ (max)} \), then
\[ l_g = 0.23 \text{ mm}. \]
If \( l_b = 20.03 \text{ (max)} \), \( l_w = 2.05 \text{ (max)} \), \( l_s = 23.9 \text{ (min)} \), then
\[ l_g = -0.23 \text{ mm (interference)}. \]

If you want assembly to be easy, no interference, then
You should specify \( l_s = 24.33 \pm 0.1 \text{ mm} \) so that the
narrowest possible distance between the fingers will still
fit the widest components.
A more accurate estimate of the gap can be found statistically, in a form of statistical analysis.

Consider a stack-up problem composed of $n$ components, each with mean length $l_i$ and tolerance $t_i$ (assumed symmetric about the mean).

In general, a length of the dependent parameter is,

$$l = l_1 \pm l_2 \pm l_3 \pm \ldots \pm l_n$$

the sign on each term depends on the structure of the device.

The standard deviation is

$$s = (s_1^2 + s_2^2 + s_3^2 + \ldots + s_n^2)^{1/2}$$

Since $s = t / 3$,

$$t = (t_1^2 + t_2^2 + t_3^2 + \ldots + t_n^2)^{1/2}$$

For the example,

$$l_g = l_s - (l_b + 2 \times l_w), \quad t_g = (t_s^2 + t_b^2 + 2 \times t_w^2)^{1/2}$$

For $l_s = 24.00 \pm 0.1$, $l_b = 20.00 \pm 0.03$, $l_w = 2.00 \pm 0.05$;

$$l_g = 24 - (20 + 2 \times 2) = 0.0 \quad \text{and} \quad t_g = (0.10^2 + 0.03^2 + 2 \times 0.05^2)^{1/2} = 0.126 \text{ mm}$$

On the average, there is no gap and the tolerance on it is 0.126 mm.
Example of Statistical Stack-Up Analysis

- For the example of Air shock-swingarm,
  \[ l_g = l_s - (l_b + 2 \times l_w), \quad t_g = (t_s^2 + t_b^2 + 2 \times t_w^2)^{1/2} \]

  For \( l_s = 24.00 \pm 0.1, l_b = 20.00 \pm 0.03, l_w = 2.00 \pm 0.05; \)
  
  \[ l_g = 24 - (20 + 2 \times 2) = 0.0 \quad \text{and} \quad t_g = (0.10^2 + 0.03^2 + 2 \times 0.05^2)^{1/2} = 0.126 \text{ mm} \]

  On the average, there is no gap and the tolerance on it is 0.126 mm.

In this problem, let’s make further assumptions:

1) When bolted, the fingers can flex up to 0.07 mm inward without undo stress on the welds to compensate for any clearance.

2) The assembly personnel can get the parts in between the fingers even if there is a 0.03 mm interference.

Then, what percentage of the assemblies will meet these requirements?

Figure shows that the probability for problems to occur during assembly is 29% (24 + 5).

How can we readjust the tolerance values?

1) Inspect each part and reworking on the numbers.
2) Determine which tolerance is most sensitive to the results using sensitivity analysis and repeat the tolerance analysis.
Sensitivity Analysis

- Technique for evaluating the statistical relationship of control parameters and their tolerances in a design problem.
  - Sensitivity analysis allows the contribution of each parameter to the variation to be easily found

- For 1-dimensional problem (air shock-swingarm):
  
  \[
  s = (s_1^2 + s_2^2 + s_3^2 + \ldots + s_n^2)^{1/2}
  \]

  For \( P_i = s_i^2 / s^2 \), where \( P_i \) is the contribution of the i-th term to the tolerance (or variance) of the dependent variable

  \[
  1 = P_1 + P_2 + \ldots + P_n
  \]

  For air shock-swingarm problem;

  \[
  P_s = (0.1 \times 0.1) / (0.126 \times 0.126) = 0.63 = 63\%
  \]

  \[
  P_b = (0.03 \times 0.03) / (0.126 \times 0.126) = 0.05 = 5\%
  \]

  \[
  P_w = (0.05 \times 0.05) / (0.126 \times 0.126) = 0.16 = 16\%
  \]

  \[
  0.63 + 0.05 + 2 \times 0.16 = 1.00
  \]

- The tolerance on the spacing has the greatest effect on the gap. Thus, the tolerance on the spacing is the most likely candidate for change.
Multi-dimensional Sensitivity Analysis

Consider a general function:

\[ F = f(x_1, x_2, x_3, \ldots, x_n) \]

\( F = \) a dependent parameter (length, volume, stress or energy) and \( x_i = \) the control parameters (usually dimensions and material properties)

For means and standard deviations \( (s_i) \),

\[ \bar{F} = f(\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_n) \]

\[ s = \left[ \left( \frac{\partial F}{\partial x_1} \right)^2 s_1^2 + \ldots + \left( \frac{\partial F}{\partial x_n} \right)^2 s_n^2 \right]^{1/2} \]

If \( \frac{\partial F}{\partial x_i} = 1 \), this SD equation becomes a linear equation
Tank Problem

For the independent parameters of \( r \) and \( l \), the mean volume is:

\[
\bar{V} = 3.1416r^2l
\]

The tolerance on these parameters can be based on what is easy to achieve with nominal manufacturing processes.

Let \( t_r = 0.03 \text{ m} \) \((s_r = 0.01)\) and \( t_l = 0.15 \text{ m} \) \((s_l = 0.05)\), then SD on this volume is:

\[
s_v = \left[ \left( \frac{\partial V}{\partial l} \right)^2 s_l^2 + \left( \frac{\partial V}{\partial r} \right)^2 s_r^2 \right]^{1/2}
\]

where

\[
\frac{\partial V}{\partial r} = 6.2830rl
\]

and

\[
\frac{\partial V}{\partial l} = 3.1416 r^2
\]

For point A, \( \partial V/\partial r = 6.61 \) and \( \partial V/\partial l = 4.60 \), so

\[
s_v = [6.61^2 \times 0.05^2 + 4.60^2 \times 0.03^2]^{1/2} = 0.239
\]

- 99.68\% (3 SD) of all the vessels built will have volumes within 0.717 m\(^3\) (3 x 0.239) of the target 4 m\(^3\).

For point B, \( \partial V/\partial r = 16 \) and \( \partial V/\partial l = 0.78 \), so

\[
s_v = [0.78^2 \times 0.05^2 + 16^2 \times 0.03^2]^{1/2} = 0.166
\]

- 99.68\% (3 SD) of all the vessels built will have volumes within 0.498 m\(^3\) of the target 4 m\(^3\).

*Reduction in variation can be achieved not by changing the tolerances on the parameters but by changing only their nominal values.

*If we can find the values of \( r \) and \( l \) that give the smallest variance on the volume, then we are employing the philosophy of robust design.
In the previous tank example, the tank with greater length had less sensitivity to the large tolerance on the length, so the tank volume varies less.

What are the most robust values for the parameters?
- It is impossible to have \( V = 4 \text{ m}^3 \), exactly due to random variations in \( r \) and \( l \).
- The best we can do is to minimize the difference between \( V \) and \( 4 \text{ m}^3 \).

The objective function to be minimized is:

\[
T = \text{target}
\]

For the tank,

\[
C = \text{variance} + \lambda \times \text{bias}
\]

\[
C = \left[ \left( \frac{\partial F}{\partial x_1} \right)^2 s_1^2 + \ldots + \left( \frac{\partial F}{\partial x_n} \right)^2 s_n^2 \right] + \lambda (F - T)
\]

For known SDs on \( r \) and \( l \), and known target \( T \)

\[
\frac{\partial C}{\partial r} = 0 = 2r(2\pi l)^2 s_r^2 + 4r^3 \pi^2 s_l^2 + \lambda 2\pi rl
\]

\[
\frac{\partial C}{\partial l} = 0 = 2l(2\pi r)^2 s_r^2 + \lambda \pi r^2
\]

\[
\frac{\partial C}{\partial \lambda} = 0 = \pi r^2 l - 4
\]

For \( s_r = 0.01 \), \( s_l = 0.05 \):

\[
r = 1.414 l \left( \frac{s_r}{s_l} \right)
\]

\[
l = \left[ \frac{2}{\pi} \left( \frac{s_l}{s_r} \right)^2 \right]^{1/3}
\]

For \( s_r = 0.01 \), \( s_l = 0.05 \):

\[
r = 0.71 \text{ m}; \quad l = 2.52 \text{ m}; \quad s_v = 0.138 \text{ m}^3
\]

*Improvement in volume variation!*

*If this SD is not small enough, we need to tighten the tolerances of \( r \) and/or \( l \).*
Summary: Robust Design

- **Step 1:** Establish the relationship between quality characteristics and the control parameters. Also define a target for the quality characteristics.

- **Step 2:** Based on known tolerances (SDs) on the control variables, generate the equation for the standard deviation of the quality characteristics.

- **Step 3:** Solve the equation for the minimum SD of the quality characteristic subject to this variable being kept on target.

\[
F = f(x_1, x_2, x_3, \ldots, x_n)
\]

\[
s = \left[ \left( \frac{\partial F}{\partial x_1} \right)^2 s_1^2 + \ldots + \left( \frac{\partial F}{\partial x_n} \right)^2 s_n^2 \right]^{1/2}
\]

Limitations on this method:
1. It is only good for design problems that can be represented by an equation.
2. The objective function used in the previous example does not allow for the inclusion of constraints in the problem. For example, if the radius had to be less than 1.0 m because of space limitations, the previous cost function would need additional terms to include.
Robust Design Through Testing

- Used when the quality characteristics cannot be represented in an equation.
  - \( V = f(r, l) \), i.e., analytically in-deterministic relationship, as compared with \( V = \pi r^2 l \), analytically deterministic relationship
  - Begin by building a tank with some best-guess dimensions and measure the volume. Repeat building a tank until we can find the right dimensions.

- Drawbacks:
  - Repetitive model building is not efficient.
  - There is no guarantee that the final design will be the most robust.

- Steps to overcome such drawbacks.
  1. Identify signals, noise, control, and quality factors (i.e., independent parameters).
  2. For each quality measure (i.e., output response) to be evaluated, recall or determine its target value and the nature of the quality loss function.
  3. Design the experiment.
  4. Take and reduce data.
  5. Analyze the results, and select new test conditions if needed.
Step 1: Identify signals, noise, control, and quality factors

Step 2: For each quality measure (output response), determine its target value and the nature of the quality loss function.

Quality loss is proportional to the mean square deviation (MSD), average difference between the output response and the target. This difference is often referred to as signal-to-noise (S/N) ratio.

<table>
<thead>
<tr>
<th>Quality Loss Function:</th>
<th>smaller-is-better</th>
<th>larger-is-better</th>
<th>Nominal-is-best</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSD</td>
<td>$\frac{1}{n} \sum_{i=1}^{n} y_i^2$</td>
<td>$\frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{y_i^2}\right)$</td>
<td>$\frac{1}{n} \sum_{i=1}^{n} \left(y_i - \bar{y}\right)^2 + \left(\bar{y} - m\right)^2$</td>
</tr>
<tr>
<td>S/N Ratio</td>
<td>$-10\log\left(\frac{1}{n} \sum_{i=1}^{n} y_i^2\right)$</td>
<td>$-10\log\left(\frac{1}{n} \sum_{i=1}^{n} y_i^2\right)$</td>
<td>$-10\log\left(\frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2\right)$</td>
</tr>
</tbody>
</table>