1. A system is described by the differential equation
   \[ \ddot{y}(t) + 8\dot{y}(t) + 16y(t) = u(t). \]
   Find the output \( y(t) \) if the input is a unit step. Assume a relaxed system.

2. For the system in problem 1, assume now the system is not relaxed. Find \( Y(s) \) as a function of \( U(s) \) and the initial conditions. Clearly mark the zero state and zero input parts of \( Y(s) \).

3. Let the transfer function of a second-order system be
   \[ H(s) = \frac{s}{s^2 + 4s + 8}. \]
   Find the impulse response \( h(t) \) for this system.

4. Consider the following model of a driver controlling an automobile. The dashed box represents a normal driver. \( u(t) \) is the desired automobile position, \( y_n(t) \) is the actual automobile position, and \( d \) and \( K \) are positive real constants.

   \[ \begin{array}{c}
   \text{Normal Driver Model} \\
   \hline
   \text{Desired Position} \quad u(t) \quad \rightarrow \quad e^{-sd} \quad \rightarrow \quad \frac{K}{s+K} \quad \rightarrow \quad \text{Car Position} \quad y(t)
   \end{array} \]

   (a) Find the automobile response \( y_n(t) \) when \( u(t) \) is a unit step input.
   (b) Now let’s modify the driver model to include the effects of driver impairment (sleepiness, distraction, alcohol, etc.). Let \( y_i(t) \) represent the car position with an impaired driver. Here is the model with an impaired driver:

   \[ \begin{array}{c}
   \text{Impaired Driver Model} \\
   \hline
   \text{Desired Position} \quad u(t) \quad \rightarrow \quad \frac{A}{s+A} \quad \rightarrow \quad e^{-sd} \quad \rightarrow \quad \frac{K}{s+K} \quad \rightarrow \quad \text{Car Position} \quad y(t)
   \end{array} \]

   Assume \( d, A, \) and \( K \) are positive real constants, with \( K > A \). Find the automobile response \( y_i(t) \) to a unit step input \( u(t) \) with the impaired driver model. Call this \( y_i(t) \) (for impaired driver).

   (c) Let \( d = 0.1 \) second, \( K = 5 \), and \( A = 2 \). Sketch the approximate shape of \( y_n(t) \) and \( y_i(t) \) for the step input. In which case does the system output respond more quickly?
5. The following model has been proposed to model the rapid-eye movement of the human eye (see figure below):

\[
T_c \frac{d\Theta_e(t)}{dt} + \Theta_e(t) = R(t),
\]

\[
T_c \frac{d\Theta_t(t - d)}{dt} + \Theta_t(t - d) = R(t),
\]

where \( \Theta_e(t) \) and \( \Theta_t(t) \) are the angular positions of the eye and the “target,” and \( T_c \) and \( d \) are known, positive constants. \( R(t) \) is the rate of firing for action potentials in the nerves to the eye muscle, \( T_c \) is a viscoelastic restoring force, and \( d \) represents the time delay through the nervous system. Assume the system is initially relaxed.

(a) Assume that the target moves with \( \Theta_t(t) = V t q(t) \), where \( V \) is a known constant. Compute \( \Theta_e(t) \).

(b) Plot both \( \Theta_t(t) \) and \( \Theta_e(t) \) for the case \( \Theta_t(t) = V t q(t) \). Interpret this answer. How does the eye track this moving target?