2.4 Color images

Human color perception

- adds wavelength of electromagnetic radiation – subjective layer on top of underlying objective physical properties
- color – a psychophysical phenomenon

- human visual system not very precise in perceiving color in absolute terms
- human notion of color is not precise
- usually relative to some widely used color standard – Ferrari red
- desire for color constancy
- computer vision – camera used as a measuring device, yields measurements in absolute quantities
- Newton (17th century) – white light is a spectral mixture
- optical prism performs decomposition
- for over 100 years later still controversial

2.4.1 Physics of color

Electromagnetic spectrum

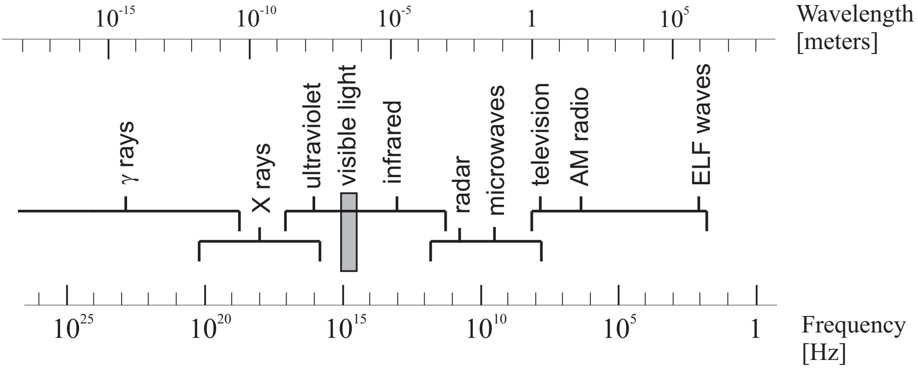


Figure 2.23: Division of the whole electromagnetic spectrum (ELF means Extremely Low Frequencies).

- only narrow section visible
- wavelength $\lambda \in (380 - -740)\,\text{nm}$

Visible colors = **spectral colors**
decomposing in a rainbow
represented as combinations of **primary colors**,
(red, green, and blue ... 700 nm, 546.1 nm, and 435.8 nm
not all colors can be synthesized as combinations of these three.

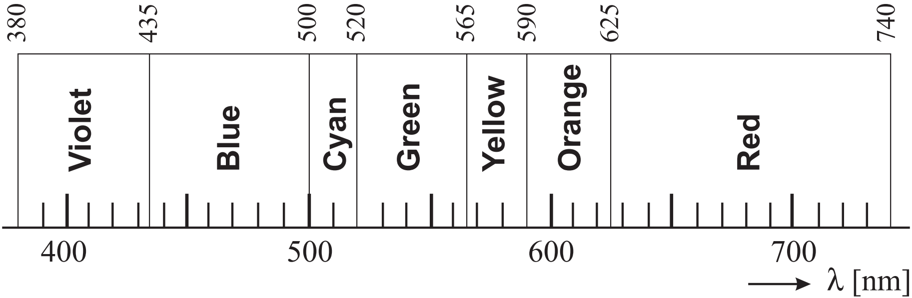


Figure 2.24: Wavelength λ of the spectrum visible to humans.

- intensity of irradiation typically not constant for different λ
- variation expressed by a **power spectrum** $S(\lambda)$

Why world in color?

- **surface reflection** – spectrum of reflected light remains the same = independent of the surface
- energy diffuses into the material and reflects randomly from the internal pigment in the matter = **body reflection** ... predominant in dielectrics as plastic or paints

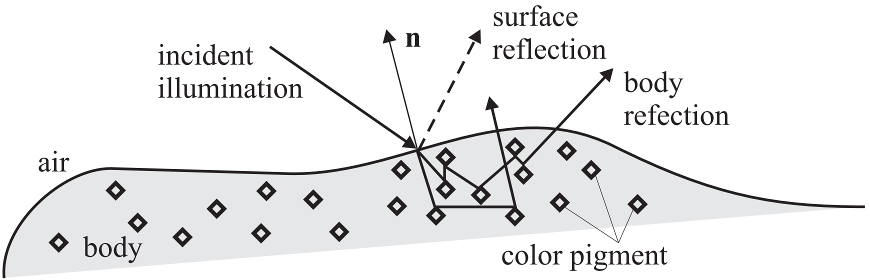


Figure 2.25: Observed color of objects is caused by certain wavelength absorptions by pigment particles in dielectrics.

- Most color capture sensors (e.g., cameras) – no direct access to color
- exception – **spectrophotometer**
  - in principle resembles Newton's prism)
  - incoming irradiation decomposed into spectral colors
  - intensity along the spectrum (changing wavelength λ) is measured in a narrow wavelength band
- **multispectral images** – intensities measured in several wavelength narrow bands
- vector describes each pixel
- each spectral band is a monochromatic image
- commonly used in remote sensing
- LANDSAT 4 – five spectral bands from near-ultraviolet to infrared

2.4.2 Color perceived by humans

- indirect color sensing (humans and some animals
- humans – three types of sensors senstive to wavelength
- $\Rightarrow$ **trichromacy**.
- humans – color sensitive receptors – **cones**
- intensity sensitive receptors – **rods** – monochromatic, for low ambient light conditions
- 3 types of cones
  - S (short) max sensitivity for $\approx 430\,\mathrm{nm}$
  - M (medium) at $\approx 560\,\mathrm{nm}$
  - L (long) at $\approx 610\,\mathrm{nm}$
  - cones S, M, L also called cones B, G and R – but this is slightly misleading
  - we do not see red solely because an L cone is activated
  - equally distributed spectrum – white
  - unbalanced spectrum – shade of color

- Mathematical modeling of photoreceptor or camera sensor:
- i – type of sensor
  $i = 1, 2, 3$, (retinal cone types S, M, L)
- $R_i(\lambda)$ ... spectral sensitivity of sensor
- $I(\lambda)$ ... spectral density of illumination
- $S(\lambda)$ ... surface patch reflectance for each wavelength

$\Rightarrow$ spectral response q_i of the i-th sensor:

$$q_i = \int_{\lambda_1}^{\lambda_2} I(\lambda)\, R_i(\lambda)\, S(\lambda)\, \mathrm{d}\lambda\,. \tag{2.17}$$

- how does vector (q_S, q_M, q_L) represent color of the surface patch?
- it does not ... Equation (2.17)
- output depends on three factors $I(\lambda)$, $S(\lambda)$ and $R(\lambda)$
- only $S(\lambda)$ related to surface patch
- $\rightarrow$ only if illumination is perfectly white, i.e., $I(\lambda) = 1$, (q_S, q_M, q_L) is an estimate of surface color

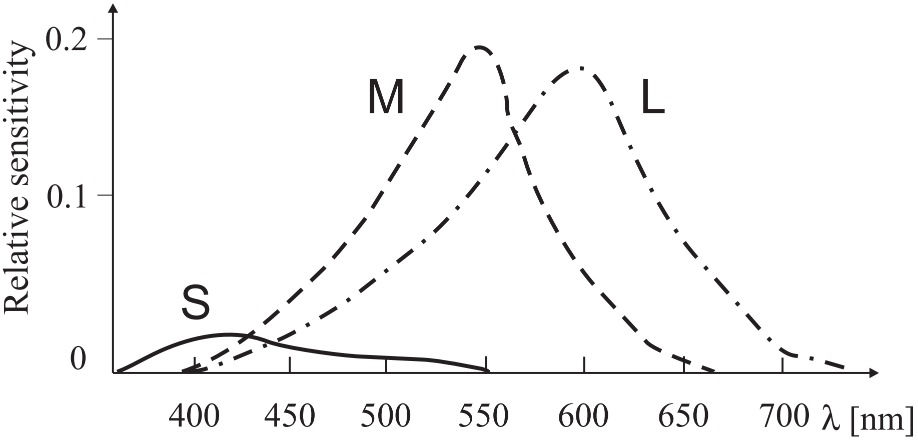


Figure 2.26: Relative sensitivity of S, M, L cones of the human eye to wavelength.

Human vision is prone to various illusions.

- Perceived color influenced by
  - objectively – spectrum of the illuminant
  - colors and scene interpretation surrounding the observed color
  - eye adaptation to changing light is slow – perception is influenced by adaptation
- but — for simplicity, let spectrum of light coming to a point on the retina fully determine the color

- color can be defined by almost any set of primaries
- $\Rightarrow$ standardized primaries and color matching functions are widely used
- **color model** is mathematical abstraction – expresses colors as tuples of numbers (typically three or four color components)
- $\Rightarrow$ **XYZ color space** ... in 1931
  - three imaginary lights X=700.0 nm, Y=546.1 nm, Z=435.8 nm
  - and color matching functions $X(\lambda)$, $Y(\lambda)$ and $Z(\lambda)$ corresponding to average human perception (through a 2° aperture)
  - standard is artificial – but, approximately – $X \approx$ red, $Y \approx$ green and $Z \approx$ blue

- XYZ color standard (CIE standard) fulfills three requirements:
  - color matching functions of XYZ color space are non-negative;
  - value of $Y(\lambda)$ coincides with brightness (luminance);
  - normalization assures that power corresponding to the three color matching functions is equal (i.e., the area under all three curves is equal).
- The resulting color matching functions are shown in Figure 2.28.
- actual color is a mixture (convex combination) of

$$c_X\, X + c_Y\, Y + c_Z\, Z\ , \tag{2.18}$$

where $0 \leq c_X,\, c_Y,\, c_Z \leq 1$ are weights (intensities) in the mixture

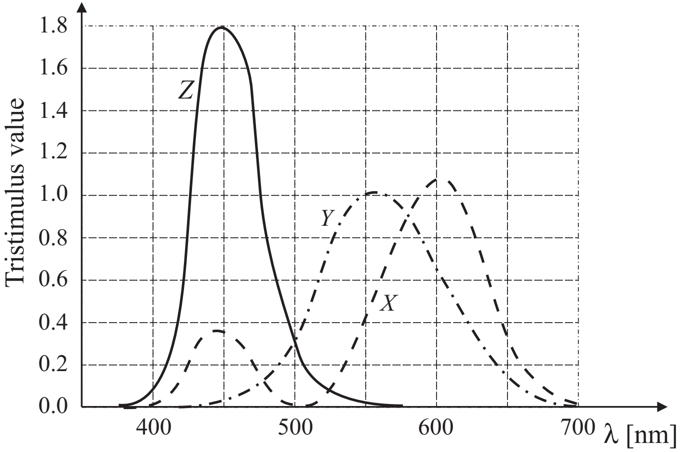


Figure 2.28: Color matching functions for the CIE standard from 1931. $X(\lambda)$, $Y(\lambda)$, $Z(\lambda)$ are color matching functions. Redrawn from [Wandell 95].

- Subspace of colors perceivable by humans = color gamut

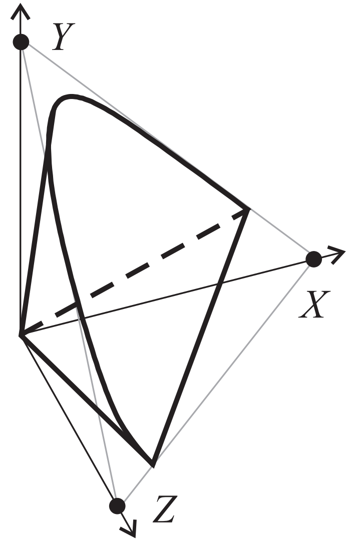


Figure 2.29: Color gamut - a subspace of the X, Y, Z color space showing all colors perceivable by humans.

- Using planar view of 3D color space ...
- projection plane passing through extremal points on all three axes, i.e., points X, Y, Z
- new 2D coordinates x, y obtained as

$$x = \frac{X}{X+Y+Z}\,, \qquad y = \frac{Y}{X+Y+Z}\,, \qquad z = 1 - x - y\,.$$

- result of this plane projection is the CIE chromaticity diagram
- horseshoe like subspace contains human-visible colors
- all human-visible monochromatic spectra map into the curved part of the horseshoe their wavelengths are shown in Figure 2.30

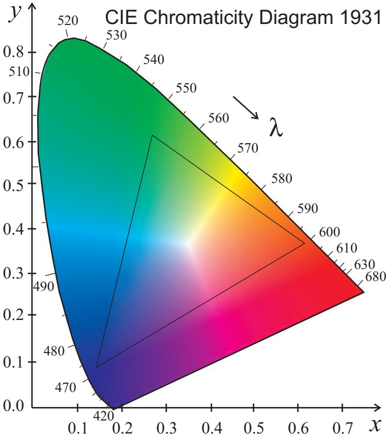
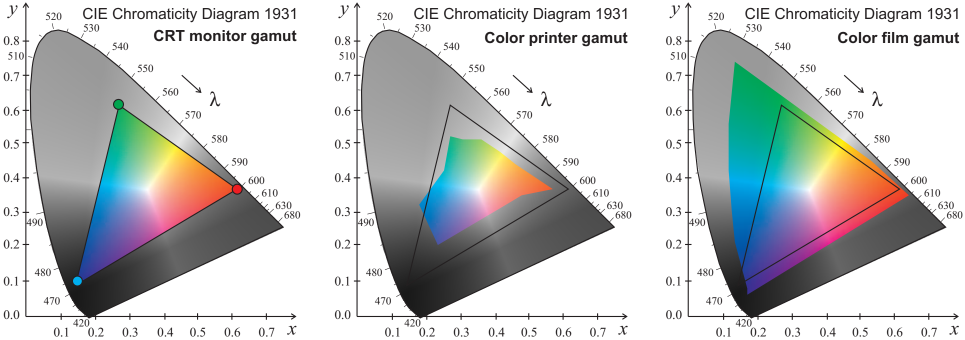


Figure 2.30: CIE chromaticity diagram is a projection of XYZ color space into a plane. The triangle depicts a subset of colors spanned by red, green, and blue. These are TV colors, i.e., all possible colors which can be seen on a CRT display. *A color version of this figure may be seen in the color inset—Plate 1.*

- display and printing devices use three selected real primary colors
- (as opposed to three syntactic primary colors of XYZ color space)
- all possible mixtures of these primary colors fail to cover the whole interior of the horseshoe in CIE chromaticity diagram
- see Figure 2.31



(a) CRT monitor (b) printer (c) film

Figure 2.31: Gamuts which can be displayed using three typical display devices. *A color version of this figure may be seen in the color inset—Plate 2.*

2.4.3 Color spaces

- different primary colors and corresponding color spaces used in practice
- these spaces can be transformed into each other
- if absolute color space is used – transformation is one-to-one and does not lose information
- since color spaces have their own gamuts, information is lost if the transformed value appears out of the gamut

- The **RGB** color space ... origin in color TV ... CRT's used
- RGB color space ... relative color standard (not absolute)
- primary colors (R–red, G–green and B–blue) mimicked phosphor in CRT luminophore
- RGB model – additive color mixing to specify which light needs to be emitted to produce a given color
- value of particular color expressed as a vector of three elements—intensities of three primary colors, recall equation (2.18)
- transformation to a different color space expressed by a transformation by a 3×3 matrix

- assume values for each primary quantized to $m = 2^n$ values
- let the highest intensity value be $k = m - 1$
- then $(0, 0, 0)$ is black, (k, k, k) is (television) white, $(k, 0, 0)$ is 'pure' red, and so on
- value $k = 255 = 2^8 - 1$ is common, i.e., 8 bits per color channel
- there are $256^3 = 2^{24} = 16,777,216$ possible colors in such a discretized space

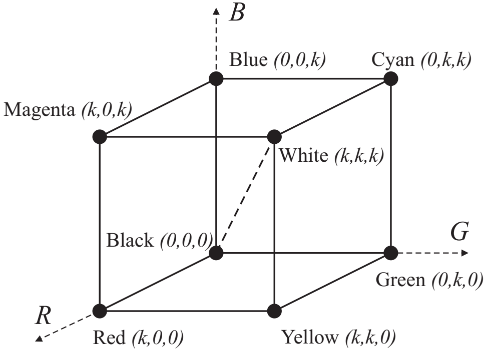


Figure 2.32: RGB color space with primary colors red, green, blue and secondary colors yellow, cyan, magenta. Gray-scale images with all intensities lie along the dashed line connecting black and white colors in RGB color space.

- RGB model – 3D color space (see Figure 2.32)

- secondary colors are combinations of two pure primaries
- additional specific instances of the RGB color model
  - sRGB
  - Adobe RGB
  - Adobe Wide Gamut RGB
- they differ slightly in transformation matrices and the gamut – e.g.,:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 3.24 & -1.54 & -0.50 \\ -0.98 & 1.88 & 0.04 \\ 0.06 & -0.20 & 1.06 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} ,$$
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.41 & 0.36 & 0.18 \\ 0.21 & 0.72 & 0.07 \\ 0.02 & 0.12 & 0.95 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} . \tag{2.19}$$

- US and Japanese color television used **YIQ** color space
- Y ... intensity
- I, Q ... color
- YIQ another example of additive color mixing
- YIQ color system – 1 luminance value + 2 chrominance values, corresponding approximately to the amounts of blue and red in the color
- YIQ color space corresponds closely to YUV color model in the PAL television norm (Australia, Europe, except France, which uses SECAM)
- YIQ color space is rotated 33° with respect to the YUV color space
- YIQ color model ... Y component provides complete monochrome information; further

- **CMY**—for Cyan, Magenta, Yellow—color model uses subtractive color mixing which – used in printing processes
- describes what kind of inks need to be applied so the light reflected from the white substrate (paper, painter's canvas) and passing through the inks produce a given color
- CMYK stores ink values for black in addition
- black color can be generated from C, M, Y components but special value black ink is advantageous
- many CMYK colors spaces are used for different sets of inks, substrates, and press characteristics (which change the color transfer function for each ink and thus change the appearance)

- **HSV** – Hue, Saturation, and Value (also known as HSB, hue, saturation, brightness)
- often used by painters – closer to their thinking and technique
- artists commonly use three to four dozen colors (characterized by the hue; technically, the dominant wavelength)
- another color is mixed from the given ones, for example, 'purple' or 'orange'
- painters want colors of different saturation, e.g., to change 'fire brigade red' to pink
- mixing 'fire brigade red' with white (and/or black) gives lower saturation

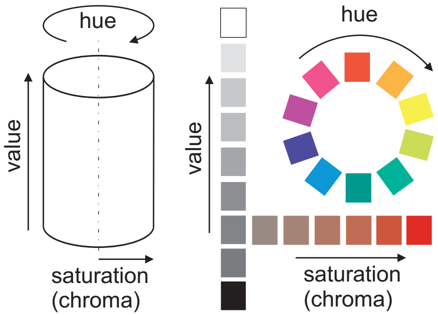


Figure 2.33: HSV color model illustrated as a cylinder and unfolded cylinder. *A color version of this figure may be seen in the color inset—Plate 3.*

- HSV decouples intensity information from color
- hue and saturation correspond to human perception
- **this representation is very useful for developing image processing algorithms**
- consider histogram equalization
  - when applied to each component of an RGB model – corrupts human sense of color
  - works well if applied to intensity component of HSV (leaving the color information unaffected)
- HSL (hue, saturation, lightness/luminance) – also known as HLS or HSI (hue, saturation, intensity) is similar to HSV
- ‘Lightness’ replaces ‘brightness’ –
  brightness of a pure color is equal to brightness of white
  lightness of a pure color is equal to lightness of a medium gray

Models	Color spaces	Applications
Colorimetric	XYZ	Colorimetric calculations
Device oriented, nonuniform spaces	RGB, UIQ	Storage, processing, coding, color TV
Device oriented, Uniform spaces	LAB, LUV	Color difference, analysis
User oriented	HSL, HSI	Color perception, computer graphics

2.4.4 Palette images

- **Palette images** (called also **indexed images**) – simple way to reduce the amount of data needed to represent an image
- link to a **lookup table** or **palette**)
- as many entries as the range of possible values in the pixel item each entry maps pixel value to color (three values, one for each of three color components)
- TIFF, PNG and GIF can store palette images

- If number of colors in input image is less than or equal to the number of entries in lookup table – all colors can be selected – no loss of information
- if number of colors exceeds number of entries in lookup table – subset of colors has to be chosen – loss of color information.
- simplest color selection – quantize color space regularly into cubes of the same size
- for 8 bit example ... $8 \times 8 \times 8 = 256$ such cubes
- not sufficient number of shades for images with any dominant color (green frog image)
- to overcome – create histograms for all three color components and quantize them to provide more shades for colors which occur in the image frequently

- **pseudocolor** – original gray-level image displayed in color

2.4.5 Color constancy

- consider image in which the same surface is seen under different illumination

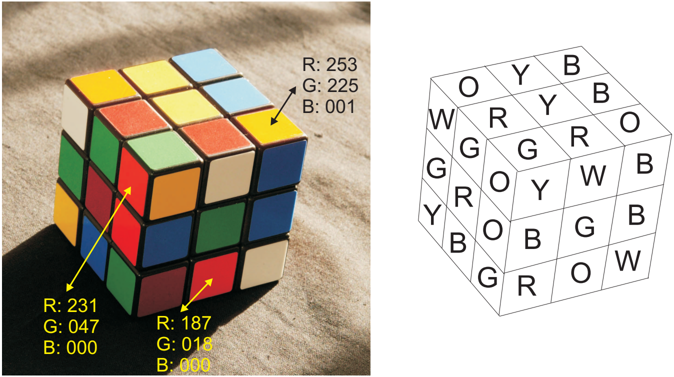


Figure 2.34: Color constancy: The Rubik cube is captured in sunlight, and two of three visible sides of the cube are in shadow. The white balance was set in the shadow area. There are six colors on the cube: R-red, G-green, B-blue, O-orange, W-white, and Y-yellow. The assignment of the six available colors to 3×9 visible color patches is shown on the right. Notice how different the same color patch can be: see *RGB* values for the three instances of orange. *A color version of this figure may be seen in the color inset—Plate 4.*

- the same surface colors may be fully or partly illuminated
- human vision system can deal with illumination changes and perceives several instances of a particular color as the same – color constancy perception
- how to equip artificial perception systems with this ability?

- recall equation (2.17)

$$q_i = \int_{\lambda_1}^{\lambda_2} I(\lambda)\, R_i(\lambda)\, S(\lambda)\, \mathrm{d}\lambda\,.$$

- color vision system has to calculate vector q_i for each pixel as if $I(\lambda) = 1$ However, spectrum $I(\lambda)$ is usually unknown
- assuming ideal case for which spectrum $I(\lambda)$ of illuminant is known – color constancy can be obtained by dividing output of each sensor by its sensitivity to the illumination
- let q_i' be the spectral response after compensation for the illuminant, $q_i' = \rho_i\, q_i$, where

$$\rho_i = 1 \Big/ \int_{\lambda_1}^{\lambda_2} I(\lambda)\, R_i(\lambda)\, \mathrm{d}\lambda\,. \tag{2.20}$$

- partial color constancy can be obtained by multiplying color responses of the three photosensors with coefficients ρ_i.
- in practice, several obstacles make this procedure intractable
  - illuminant spectrum $I(\lambda)$ is not known and can only be guessed indirectly from reflections in surfaces
  - only approximate spectrum is expressed by the spectral response q_i of the i-th sensor
  - $\Rightarrow$ color constancy problem is ill-posed and cannot be solved without making additional assumptions about the scene
- it can be assumed that average color of the image is gray $\Rightarrow$ it is possible to scale the sensitivity of each sensor type until the assumption becomes true
- this results in insensitivity to the color of the illumination
- this type of color compensation is often used in automatic white balancing in video cameras

- another common assumption – brightest point in the image has the color of the illumination – true when scene contains specular reflections which have the property that the illuminant is reflected without being transformed by the surface patch

The problem of color constancy is further complicated by the perceptual abilities of the human visual system

- Humans have quite poor quantitative color memory, and also perform color adaptation
- the same color is sensed differently in different local contexts

2.5 Cameras: an overview

2.5.1 Photosensitive sensors

Photosensitive sensors commonly found in cameras:

Sensors based on photo-emission principles – explore photoelectric effect

- external photon provokes emission of a free electron
- phenomenon exhibited most strongly in metals
- used in photomultipliers and vacuum tube TV cameras

Sensors based on photovoltaic principles – in semiconductors

- energy of a photon causes an electron to leave its valence band and changes to a conduction band
- quantity of incoming photons affects macroscopic conductivity
- excited electron = source of voltage = results in electric current
- current directly proportional to the amount of incoming energy (photons)
- photodiode
- avalanche photodiode (similar to photomultiplier; also amplifies noise, used, e.g., in night vision cameras)
- photoresistor
- Schottky photodiode

- two types of semiconductor photoresistive sensors used widely
- CCDs (charge-coupled devices)
- CMOS (complementary metal oxide semiconductor)
- neither categorically superior to the other

- CCD sensor
  - every pixel's charge is transferred through just one output node to be converted to voltage, buffered, and sent off-chip as an analog signal
  - entire pixel area can be devoted to light capture
- CMOS sensor
  - each pixel has its own charge-to-voltage conversion
  - sensor often includes amplifiers, noise-correction, and digitization circuits $\rightarrow$ chip outputs (digital) bits
  - $\Rightarrow$ increase in design complexity
  - reduction of area available for light capture

- CCD sensor includes a Schottky photodiode and a field-effect transistor
- photon falling on the junction of the photodiode liberates electrons from the crystal lattice and creates holes, resulting in the electric charge that accumulates in a capacitor
- collected charge directly proportional to the light intensity and duration of its falling on the diode
- sensor elements arranged into a matrix-like grid of pixels—a CCD chip
- charges accumulated by the sensor elements are transferred to a horizontal register one row at a time by a vertical shift register
- charges are shifted out in a bucket brigade fashion to form the video signal

Three inherent problems of CCD chips.

- Blooming effect = mutual influence of charges in neighboring pixels (Anti-blooming technology helps greatly by now).
- Impossible to address directly individual pixels in the CCD chip (read out through shift registers is needed)
- Individual CCD sensor can accumulate $\sim$30–200,000 electrons
  - inherent CCD noise at the level of 20 electrons
  - signal-to-noise ratio (SNR) in the case of a cooled CCD chip is SNR = $20 \log(200000/20)$
  - logarithmic noise 80 dB
    CCD sensor can cope with four orders of magnitude of intensity in the best case
  - drops to approximately two orders of magnitude with common uncooled CCD cameras
  - range of incoming light intensity variations is usually higher

COMPARE ...

- human eye – range of nine orders of magnitude (if time for adaptation is provided)
- but – CCD cameras have high sensitivity (are able to see in darkness)
- low levels of noise
- CCD elements are common, also in digital photo cameras

Matrix-like sensors based on CMOS technology

- CMOS technology used in processors and memories
- $\rightarrow$ mass production leads to low prices
- photosensitive matrix-like element can be integrated to the same chip as the processor and/or operational memory
- $\Rightarrow$ 'smart cameras' in which the image capture and basic image processing is performed on the same chip

- CMOS cameras – range of sensed intensities = about 4 orders of magnitude
- high speed of read-out (about 100 ns)
- random access to individual pixels
- ... disadvantage – higher level of noise (one degree of magnitude higher)

2.5.2 A monochromatic camera

Analog cameras

- complete TV signal ... light intensity, horizontal and vertical synchronization pulses
- allows row by row display
- interlaced or non-interlaced lines
- 60 half-frames per second, 525 lines (USA and Japan)
- 50 half-frames per second, 625 lines (Europe)
- analog cameras require a digitizer card (a frame grabber) to be plugged in to the computer.

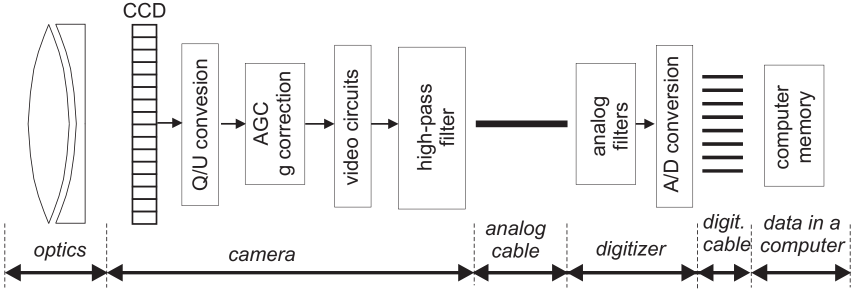


Figure 2.35: Analog CCD camera. Q/U – photon energy to voltage conversion. ACG - automatic gain control – mus tbe switchable for measurement purposes. γ correction - needed for CRT displays, not needed for LCD.

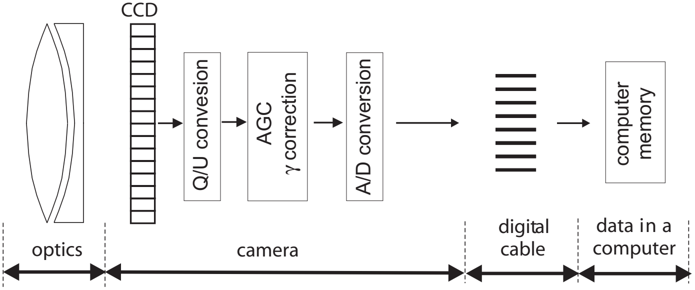


Figure 2.36: Digital CCD camera. Firewire or USB connection to computer is typical.

	Analog cameras		Digital cameras
+	Cheap.	+	Cheap webcams. Dropping price for others
+	Long cable possible (up to 300 m).	−	Shorter cable ($\approx$ 10 m for Firewire). Kilometers after conversion to optical cable Any length for Internet cameras.
−	Multiple sampling of a signal.	+	Single sampling.
−	Noisy due to analog transmission.	+	No transmission noise.
−	Line jitter.	+	Lines are vertically aligned.

2.5.3 A color camera

Electronic photosensitive sensors are monochromatic

- Color filters → three different images in succession
  - used only in precise laboratory measurements
  - impractical and impossible for any subject involving motion
- Use color filter array on a single sensor (widely used)
  - each pixel covered with individual filter – on a cover glass on the chip package (hybrid filter) or directly on the silicon (monolithic filter)
  - each pixel captures only one color
  - ⇒ color resolution one third of the geometric resolution
  - full color values for each pixel can be interpolated from pixel values of the same color in local neighborhood.
  - human eye most semnsitive to green ... more green pixels (see Figure 2.37)

G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G

Figure 2.37: Bayer filter mosaic for single chip color cameras.

- Incoming light split into several color channels using prism-like devices
  - multiple-chip cameras use color filters to split incoming light into separate color channels
  - photosensors are simple and preserve spatial resolution
  - aligning and registering the sensors to the color splitter to the prism requires high precision

2.6 Summary

- **Basic concepts**
  - A 2D image gray-scale image is represented by a scalar function $f(x, y)$ of two variables which give coordinates in a plane.
  - In many cases, a 2D image is formed as the result of a projection of a 3D scene into 2D.
  - The domain of the digitized image is a limited discrete grid the coordinates of which are natural numbers. The range of the digitized image is a limited discrete set of gray values (brightnesses). A pixel represents the elemental part of an image.
- **Image digitization**
  - Digitization (sampling) of an image can be seen as a product of a sampling function and a continuous image function.
  - Usually the grid consists of regular polygons (squares or hexagons). The second aspect of sampling is setting the distance between the sampling points (the smaller sampling distance the higher the resolution of the image).
  - Gray level quantization governs the appearance of shading and false contour. A human is able to recognize about 60 gray levels at most. Images containing only black and white pixels are called the binary.
- **Digital image properties**

- The neighborhood relation of a pixel has to be defined to be able to represent discrete geometry.
- A function providing distance between two pixels has to be established—there are several definitions used. The most commonly used is 'city block', 'chessboard', and the Euclidean distance used in everyday life. If the neighborhood relation is set for a grid then a raster is obtained.
- Given a raster, topological properties are induced. These properties are based on the relation 'being contiguous' and lead to concepts of region, background, hole, and region border. The convex hull of a region is the minimal convex subset containing it.
- 4-neighborhoods and 8-neighborhoods lead to 'crossing line' paradoxes which complicate basic discrete geometry algorithms. However, there exist solutions to these paradoxes for both binary and grey-level images.
- The distance transform (chamfering) of a binary image provides the distance from each pixel to the nearest non-zero pixel. There is a computationally effective two-pass algorithm to compute this, the complexity of which depends linearly on the number of pixels.
- The brightness histogram is a global descriptor of the image giving the estimate of the probability density that a pixel has a given brightness.
- Human visual perception is vulnerable to various illusions. Some of the properties of human perception of images as perceptual grouping are inspirations for computer vision methods.

- Live images as any other measurements or observations are always prone to noise. It is possible to assess the noise extent quantitatively using, e.g., signal-to-noise ratio.
- White, Gaussian, impulse, and salt-and-pepper noise are common models.

• **Color images**

- Human color perception is a subjective psychophysical layer on top of underlying objective physical properties—the wavelength of electromagnetic radiation.
- Three types of sensors receptive to the wavelength of incoming irradiation have been established in humans. Color sensitive receptors on the human retina are cones. The other light sensitive receptors on the retina are rods which are dedicated to sensing monochromatically in low ambient light conditions. Cones are categorized into three types based on the sensed wavelength range, approximately corresponding to red, green and blue.

• **Cameras**

- Most cameras use either CCD or CMOS photosensitive elements, both using photovoltaic principles. They capture brightness of a monochromatic image.
- Cameras are equipped with necessary electronics to provide digitized images. Color cameras are similar to monochromatic ones and contain color filters.

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