Course Syllabus

1. Color
2. Camera models, camera calibration
3. Advanced image pre-processing
   - Line detection
   - Corner detection
   - Maximally stable extremal regions
4. Mathematical Morphology
   - binary
   - gray-scale
   - skeletonization
   - granulometry
   - morphological segmentation
   - Scale in image processing
5. Wavelet theory in image processing
6. Image Compression
7. Texture
8. Image Registration
   - rigid
   - non-rigid
   - RANSAC
Quiz

Object = black

original

?  

?
13.5 Skeletons and object marking

13.5.1 Homotopic transformations

- transformation is homotopic if it does not change the continuity relation between regions and holes in the image.

- this relation is expressed by homotopic tree
  - its root ... image background
  - first-level branches ... objects (regions)
  - second-level branches ... holes
  - etc.

- transformation is homotopic if it does not change homotopic tree
Homotopic Tree
Homotopic Tree
Homotopic Tree
Quiz: Homotopic Transformation

• What is the relation between an element in the ith and i+1th levels?
13.5.2 Skeleton, maximal ball

- skeletonization = **medial axis transform**
- ‘grassfire’ scenario
- A grassfire starts on the entire region boundary at the same instant – propagates towards the region interior with constant speed
- **skeleton** \( S(X) \) ... set of points where two or more fire-fronts meet

**Figure 13.22**: Skeleton as points where two or more fire-fronts of grassfire meet.

- Formal definition of skeleton based on maximal ball concept
- **ball** \( B(p, r) \), \( r \geq 0 \) ... set of points with distances \( d \) from center \( \leq r \)
- ball \( B \) included in a set \( X \) is **maximal** if and only if there is no larger ball included in \( X \) that contains \( B \)
Figure 13.23: Ball and two maximal balls in a Euclidean plane.
plane $\mathbb{R}^2$ with usual Euclidean distance gives unit ball $B_E$

three distances and balls are often defined in the discrete plane $\mathbb{Z}^2$

if support is a square grid, two unit balls are possible:

- $B_4$ for 4-connectivity
- $B_8$ for 8-connectivity

skeleton by maximal balls $S(X)$ of a set $X \subseteq \mathbb{Z}^2$ is the set of centers $p$ of maximal balls

$$S(X) = \{ p \in X : \exists r \geq 0, B(p, r) \text{ is a maximal ball of } X \}$$

this definition of skeleton has intuitive meaning in Euclidean plane

skeleton of a disk reduces to its center

skeleton of a stripe with rounded endings is a unit thickness line at its center

etc.

Figure 13.24: Unit-size disk for different distances, from left side: Euclidean distance, 6-, 4-, and 8-connectivity, respectively.
• skeleton by maximal balls – two unfortunate properties
• does not necessarily preserve homotopy (connectivity)
• some of skeleton lines may be wider than one pixel
• skeleton is often substituted by sequential homotopic thinning that does not have these two properties
• dilation can be used in any of the discrete connectivities to create balls of varying radii
• \( nB = \text{ball of radius } n \)
  \[ nB = B \oplus B \oplus \ldots \oplus B \]
• skeleton by maximal balls ... union of the residues of opening of set \( X \) at all scales
  \[ S(X) = \bigcup_{n=0}^{\infty} \left( (X \ominus nB) \setminus ((X \ominus nB) \circ B) \right) \]
• trouble: skeletons are disconnected - a property is not useful in many applications
• **homotopic skeletons** that preserve connectivity are preferred

**Figure 13.25:** Skeletons of rectangle, two touching balls, and a ring.
13.5.3 Thinning, thickening, and homotopic skeleton

- hit-or-miss transformation can be used for **thinning** and **thickening** of point sets
- image $X$ and a composite structuring element $B = (B_1, B_2)$
- notice that $B$ here is not a ball
- **Thinning**
  
  $X \ominus B = X \setminus (X \otimes B)$

- **Thickening**

  $X \odot B = X \setminus (X \otimes B)$

- thinning – part of object boundary is subtracted by set difference operation
- thickening – part of background boundary is added

- Thinning and thickening are dual transformations

  $(X \odot B)^c = X^c \ominus (B_2, B_1)$
• Thinning and thickening often used sequentially
• Let \( B = \{ B_{(1)}, B_{(2)}, B_{(3)}, \ldots, B_{(n)} \} \) denote a sequence of composite structuring elements \( B_{(i)} = (B_{i_1}, B_{i_2}) \)
• **Sequential thinning** – sequence of \( n \) structuring elements
  \[
  X \ominus B = \left( \left( (X \ominus B_{(1)}) \ominus B_{(1)} \right) \ldots \ominus B_{(n)} \right)
  \]
• **sequential thickening**
  \[
  X \oslash B = \left( \left( (X \oslash B_{(1)}) \oslash B_{(1)} \right) \ldots \oslash B_{(n)} \right)
  \]
• several sequences of structuring elements \( \{ B_{(i)} \} \) are useful in practice
• e.g., permissible rotation of structuring element in digital raster (e.g., hexagonal, square, or octagonal)
• These sequences are called the **Golay alphabet**
• composite structuring element – expressed by a single matrix
• “one” means that this element belongs to \( B1 \) (it is a subset of objects in the hit-or-miss transformation)
• “zero” belongs to \( B2 \) and is a subset of the background
• \(*\) ... element not used in matching process = its value is not significant
• Thinning and thickening sequential transformations converge to some image — the number of iterations needed depends on the objects in the image and the structuring element used

• if two successive images in the sequence are identical, the thinning (or thickening) is stopped
Sequential thinning by structuring element $L$

- thinning by $L$ serves as homotopic substitute of the skeleton;
- final thinned image consists only of lines of width one and isolated points

- structuring element $L$ from the Golay alphabet is given by

  $$L_1 = \begin{bmatrix} 0 & 0 & 0 \\ d & 1 & d \\ 1 & 1 & 1 \end{bmatrix}, \quad L_1 = \begin{bmatrix} d & 0 & 0 \\ 1 & 1 & 0 \\ d & 1 & d \end{bmatrix}$$

- (The other six elements are given by rotation).

Original | after 5 iteration | final result
Sequential thinning by structuring element $E$

- Structuring element $E$ from the Golay alphabet is given by

$$E_1 = \begin{bmatrix} d & 1 & d \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 0 & d & d \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- Less jagged skeletons

Original                              skeleton
Axial line detection using Distance transform

A point $p$ is an axial point if there is no point $p'$ such that a shortest path from $p'$ to the boundary passes through $p$.

A point $p$ is an axial point if there is no point $q$ in the neighborhood of $p$ such that

$$DT(q) = DT(p) + |p - q|$$
Axial line detection using Distance transform
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