57:020 Mechanics of Fluids and Transfer Processes

OVERVIEW OF UNCERTAINTY ANALYSIS

INTRODUCTION

Measurement is the act of assigning a value to some physical variable. In the ideal measurement, the value assigned by the measurement would be the actual value of the physical variable intended to be measured. However, measurement errors bring on an uncertainty in the correctness of the value resulting from the measurement. To give some measure of confidence to the measured value, measurement errors must be identified, and their probable effect on the result estimated. Uncertainty is simply an estimate of a possible value for the error in the reported results of a measurement. Uncertainty analysis provides a structured and rational framework of evaluating the significance of the scatter (precision errors) and trends (potentially associated with bias errors) in the data.

The purpose of this presentation is to provide a general view on the type of engineering experiments and their requirements with respect to data quality. The terms *data quality* and *uncertainty* are commonly used interchangeably to reinforce the concept that intelligent design, execution, and documentation of an experiment add great value to the results. In the following an overview of uncertainty analysis methodology is presented, and its application in the different phases of an experimental program is discussed.

A companion write-up presents the uncertainty assessment methodology (Stern et al., 1999). The uncertainty analysis methodology adopted herein is a direct outgrowth of the AIAA S-071-1995 Standard (AIAA, 1995). Stern et al. (1999) provide comprehensive guidelines for application/integration of uncertainty assessment methodology into the test process and documentation of results. At this point it is assumed that the most basic methods used to understand and quantify finite data sets is familiar to the reader. For a quick reference on the probability and the statistical concepts involved in data reduction and analysis Figliola and Beasley (1991) will suffice.

ERRORS AND UNCERTAINTIES

Errors can be considered to be composed of two components: a precision component and a bias component. The precision error (e) is the random error and will have a different value for each measurement. The bias error (b) is the fixed, systematic, or constant error. The effects of such errors on multiple readings of a variable X are illustrated in Figure 1. The degree of inaccuracy or the total measurement error (d) is the difference between the measured value and the true value. An accurate value is one with small bias and precision errors.

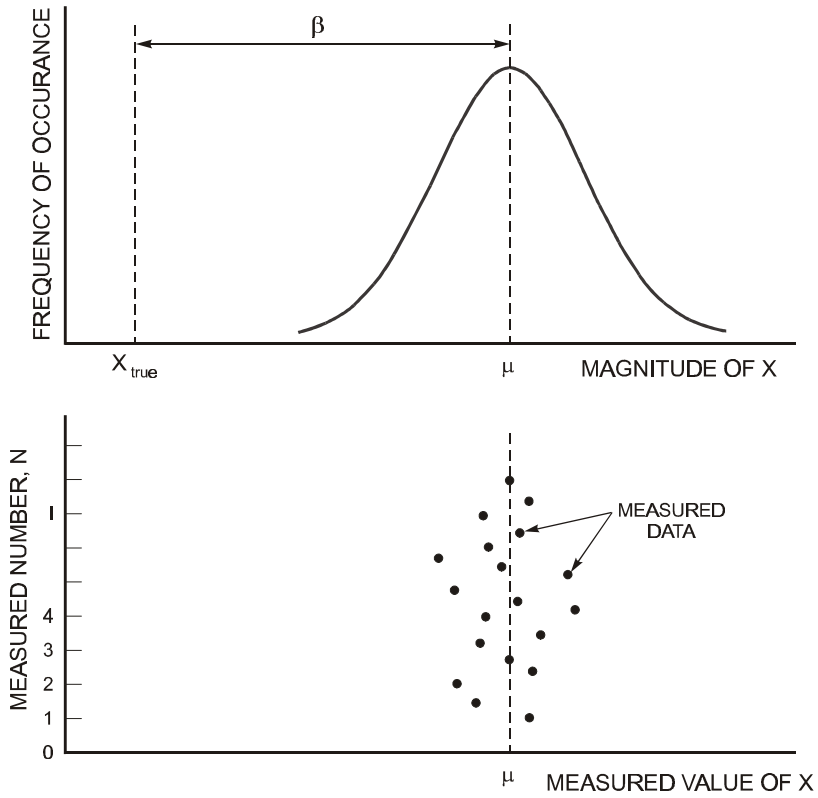

Figure 1. Errors in the measurement of a variable *X*

Only in rare circumstances the true value of a quantity is known. For example, true values of standard measurement quantities (e.g., mass, length, time, volts, etc.) reside in national standards laboratories. In general, each measurement system that is used to measure the value of an individual variable is influenced by a large number of elemental error sources. The total error is a combination of these errors. Thus, one is forced to estimate precision error (termed precision limit) and bias error (termed bias limit). The uncertainty in a given measurement of a physical quantity X is the estimate for the total error U_x obtained as a weighted average of the bias and precision limits for each reported result. The estimation of the uncertainty is made with a C% confidence. The best estimate of the true value sought in a measurement is provided by its sample mean value and the uncertainty in that value (estimated with a confidence C%)

$$X = \overline{X} \pm U_x$$

In general, the uncertainty of a quantity is a function of the value of that quantity. However, it is common practice to quote the same value of uncertainty for a range of values of the quantity, e.g., percent of full scale of an instrument. In this presentation, all estimates are assumed to be made at a 95-percent confidence level (C%), meaning that the true value of the quantity is expected to be within the $\pm U$ interval about the experimentally determined value 95 times out of 100.

Error Sources

Calibration errors. Elemental errors can enter the measurement system during the process calibration. There are two principal sources: (1) the bias and precision errors in the standard used in the calibration and (2) the manner in which the standard is applied to the measuring system or system component.

Data acquisition errors. All errors due to the actual act of measurement are referred to as data acquisition errors. These errors include: actual sensor and instrument error, changes or unknowns in power settings and environmental conditions, and sensor installation effects on the measurand. Also, the measured variable temporal and spatial variations contribute errors through the unknowns of finite statistics.

Data Reduction errors. In general measured values of several variables are combined in a data reduction equation to obtain the value of the final result. The resolution of computational operations required to reduce the data into result is a common contribution to the total error. The use of curve fits and correlations with their associated unknowns also introduces data reduction errors into the reported test results.

Conceptual bias errors. In some of the cases we are not actually measuring the variable needed in the data reduction equation. For example, in fluid flow in a pipe, we might need the average velocity but only be able to measure the velocity at one point with the equipment available. The relationship between this single measurement of velocity and the average velocity must be inferred from auxiliary information and included as a contributor to the bias in the uncertainty calculation.

Uncertainty in property values. In many experiments, values for some of the variables in the data reduction equation are not measured, but rather are found from reference sources. This is often the case for material properties, which are typically tabulated. Whether we enter the table (100 times over a month) to obtain a property value or use a curve fit equation that represents the table, we will always obtain the same particular property value. This value is not the true value, it is the “best estimate” based on experimental data and has an uncertainty associated with it. It is assumed that all of the errors (both precision and bias) in the experimental property data are “fossilized” into a bias error. In practical terms, this generally means that we have to estimate an uncertainty band based on the data scatter from different experiments

Summary of Uncertainty Assessment Methodology

The methodology for estimating the uncertainties in measurements, and in the experimental results calculated from them, must be structured to combine statistical and engineering concepts in a manner that can be systematically applied to each step in the data uncertainty assessment determination.

The uncertainty assessment methodology is summarized in Figure 2 (adapted from AIAA, 1995). For each experimental result, the data reduction equation must be determined. Once this has been done, the X_i's that must be considered are known, and the sources of uncertainty for each X_i should be identified. (Note that a math model for a correction, such as for blockage or wall interference effects, is an X_i whose uncertainty must also be considered.)

Once the sources of uncertainty have been identified, their relative significance should be established. This is often done using order of magnitude estimates of the sources. As a "rule of thumb" for a given X_i, those uncertainties sources that are smaller that 1/4 or 1/5 of the largest

sources are usually considered negligible. Resources can then be concentrated on obtaining estimates of those uncertainties of most importance.

For each X_i, estimated of the precision limit and the bias limit are then made. In most experimental tests, it is generally not cost effective or necessary to try to estimate precision limits at the elemental error source level. It is far more effective to estimate the precision of the measurement systems if multiple results at same set point are available (end-to-end procedure). Of course, if one encounters unacceptably large precision limits, the elemental sources' contributions must be examined to see which need to be (or can be) improved. It is generally easiest to obtain an estimate of the bias limit for X_i by estimating the bias limits of the significant elemental sources.

The precision limit, bias limit, and overall uncertainty for the experimental result, *r*, are then estimated.

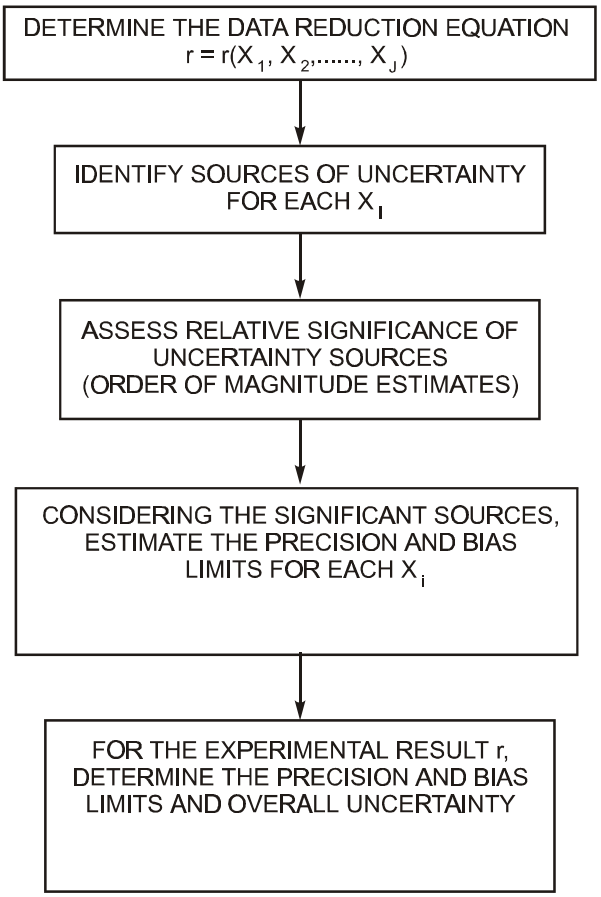

Figure 2. Summary of the uncertainty assessment methodology

Reporting Uncertainties

For each experimental result, the bias limit, precision limit, and overall uncertainty should be reported. For situations in which the large sample assumption is not applicable, the small sample methodology and coverage factor used should be reported and discussed. If outliers are rejected, the circumstances and rationale used in rejecting them should reported. Details of the uncertainty assessments should be documented either in an appendix to the primary test report or in a separate document that can be referenced in the primary test report.

EXPERIMENTS

Experiments have a wide range of purposes. Of particular interest are fluids engineering experiments conducted for science and technological advancement; research and development; design, test, and evaluation; and product liability and acceptance.

Timewise and Sample-to-Sample Experiments

When the measured quantity has a variability unrelated to the bias and precision errors inherent in the measurement system, it is helpful to distinguish between two type of experiments: timewise experiments and sample-to-sample experiments. Timewise experiments are those in which a given entity is tested, either as function of time or at some steady-state condition in which data are taken over some period of time. Examples would be testing the performance characteristics of a given engine, determining the friction factor for a given pipe over a range of Reynolds numbers.

Sample-to-sample experiments are those in which some characteristics is determined for sample after sample (realization), often with the variability from sample to sample being significant. In this case, sample identity can be viewed as analogous to time in a timewise experiment. Examples would be determining the heating value of a certain type coal, determining some physical characteristics of a manufactured product for quality control purposes.

Repetition and Replication

The word repetition will be used in its common sense, that is, to mean that something is repeated. Replication is a repetition carried out in a very specific manner. The reason for doing this is that different factors influence the errors in a series of measurements depending on how the repetition is done.

It is convenient to define three levels of replication: zeroth order, first order, and Nth order. At the zeroth-order replication in a timewise experiment level (one experiment, multiple measurements, same instrumentation), the process being measured is hypothesized to be absolutely steady. This allows only the variations inherent in the measuring system itself to contribute to precision errors. In a sample-to sample type of experiment, this corresponds to consideration of a single fixed sample.

In a first-order replication level in a timewise experiment (multiple experiments, multiple measurements, same instrumentation), we hypothesize that the time runs but all instrument identities are fixed. At this level of replication, the variability of the experimental measurements is influenced by all of the factors that contribute to unsteadiness during repeated trials with the experimental apparatus (e.g., variations of humidity, ambient pressure, our inability to exactly reproduce a set point with the experimental apparatus, etc. can influence the random portion of

the experimental uncertainty). In considering uncertainties at the first-order replication level in a sample-to-sample experiment, we imagine all instruments remain the same as sample after sample are tested.

The Nth-order replication level (multiple experiments, multiple measurements, multiple instrumentation) is used when we have to specify where the true value lies relative to our measurements. Such Nth order estimates of uncertainty include the first-order replication level estimates of precision errors together with all of the bias errors that influence our measurements. For timewise experiments, at Nth order replication both time and instrument identities are considered to vary. At this level, for each reading each instrument is considered to have been replaced by another of the same type (i.e., the bias error associated with a particular instrument becomes a random variable).

General and Detailed Uncertainty Analysis

In most of the experimental studies we measure the values for several variables and combine these in a data reduction equation to obtain the value of the desired result. The measurements of the variables have uncertainties associated with them, and the values of the material properties that we obtain from reference sources also have uncertainties. How do the uncertainties in the individual variables propagate through a data reduction equation at a result? This is the key question answered by the uncertainty analysis.

In the planning phase of an experimental program, the approach we use considers only the general, or overall, measurement uncertainties and not the details of the bias and precision components. This approach will be termed *general uncertainty analysis*. It makes sense to consider only the overall uncertainty in each measured variable at this stage rather than worry about which part of the uncertainty will be to bias and which part will be due to precision errors. In this stage the particular equipment and instruments will not have been chosen, therefore the bias errors are zero and the uncertainty in the results comprises only the precision errors. Of course, at this stage there are usually no samples from which to compute statistical estimates of the precision errors, therefore range of uncertainties in the variables are assumed.

Once past the planning phase in an experimental program, it is desirable and useful to consider the details of the bias and precision errors in each measured variable, and the propagation of the bias and the precision errors into the result. This is termed *detailed uncertainty analysis* as the details of the bias and precision components of the uncertainties are considered. The bias is a fixed error that can be reduced by calibration. However, the precision error is a variable error that can be reduced by the use of multiple readings. This differing behavior of the two components of the uncertainty makes it desirable and necessary to consider the components separately.

Phases of an Experimental Program

A general experimental program can be divided in the following experimental phases (Coleman and Steel, 1989): planning (evaluate various approaches), design (specify instrumentation and details of the experimental apparatus), construction (assembly of individual components and calibration of instruments), debugging (trial runs), execution (experimental runs and data acquisition, recording and storage), data analysis (answer original questions), and reporting of the results (data and conclusions presented). Use of uncertainty analysis in each of these phases will help to ensure maximum return for the time, effort, and financial resources

invested. An overview of some of the uses of uncertainty analysis in the different phases of an experimental program is given in Table 1.

In the planning phase general uncertainty analysis is used to ensure that a given experiment can successfully answer the question of interest. Some decisions in the preliminary design of an experiment can be made based on the results of a general uncertainty analysis.

Once past the planing and preliminary design phases, the effects of bias errors and precision errors are considered separately using the techniques of detailed uncertainty analysis. This means that estimates of bias and precision limits will be made and used in the design phase, then in the construction, debugging, execution, and data analysis phases, and finally in the reporting phase of an experiment as shown in Table 1. As an experimental program progresses there will be more information available to estimate the bias limit and precision limits. This means that these estimates are changed during the experimental phases. Bias limits and precision limits must be estimated using the best information available at the time. Lack of information is no excuse for not doing an uncertainty analysis in the early phases of an experimental program; it is simply a reason why estimates may not be as good as they will be later in the program.

Table 1. Uncertainty analysis in experimentation

Phase of Experiment	**Type of uncertainty analysis**	**Uses of uncertainty analysis**
Planning	General	Choose experiment to answer a question; preliminary design
Design	Detailed	Choose instrumentation (zeroth order estimates); detailed design (Nth order estimates)
Construction	Detailed	Guide decisions on changes
Debugging	Detailed	Verify and qualify operation; first order and Nth order comparisons
Execution	Detailed	Balance checks and monitoring operation of apparatus; choice of test points run
Data Analysis	Detailed	Guide to choice of analysis techniques
Reporting	Detailed	Bias limits, precision limits and overall uncertainties reported

The manner in which these estimates are used can differ in timewise and sample-to-sample experiments. In the early stage of the design phase of a program, estimates of the bias limits and precision limits at the zeroth-order replication level are useful in choosing the instrumentation and measurement systems. For timewise experiments, this means making estimates while hypothesizing a totally steady process environment. For sample-to-sample experiments, it means making the estimates assuming a single, fixed sample. The zeroth-order bias and precision limit estimates indicate the “best case” for a given measurement system in both types of experiment.

When we move beyond this stage in a timewise experiment, we make estimates at the first-order and Nth order levels. Here we consider all the factors that will influence the bias and precision errors in the experiment. At the first-order replication level, we are interested in the variability of the experimental results for a given experimental apparatus. The descriptor for this variability is P_r, the precision limit of the result. In a timewise experiment, comparison of the

estimated P_r and the observed scatter in the results from multiple trials at a given set point of the experimental apparatus is useful in debugging phase. In a sample-to-sample experiment, first-order estimates of P_r made before multiple samples are tested are often not very useful, since the variation from sample to sample is usually unknown and is one of the things to discover with the experiment. After multiple samples have been tested, the difference (in a root-mean-square sense) between the calculated P_r from the multiple results and the zeroth-order precision limit estimate can be used as an estimate of the precision contribution due to sample-to-sample variability.

In asking questions and making comparisons at the Nth order replication level, we are interested in the interval within which "the truth" lies. This interval is described by U_r, the overall uncertainty in the result, which is found by combining the first order precision limit P_r and the bias limit B_r. Comparisons of experimental data with theoretical results or with data from other experiments should be made at the Nth order replication level.

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