53:071 Principles of Hydraulics
Laboratory Experiment #3

ANALYSIS OF OPEN-CHANNEL FLOW TRANSITIONS USING THE SPECIFIC ENERGY DIAGRAM

Principle

Adaptation of the Bernoulli equation to open-channel flows under the assumption of small energy losses and hydrostatic pressure distribution leads to the specific energy diagram that facilitates analysis of flow behavior at channel transitions.

Introduction

Bernoulli equation applied to the free surface in an open channel (see Figure 1.a) under the assumption that the pressure distribution is hydrostatic can be written as

$$\left(z + y + \frac{V^2}{2g}\right)_1 = \left(z + y + \frac{V^2}{2g}\right)_2 \qquad (1)$$

where V is the mean channel flow, g is the gravitational acceleration, and the other notations are as shown in Figure 1.a

a) b)

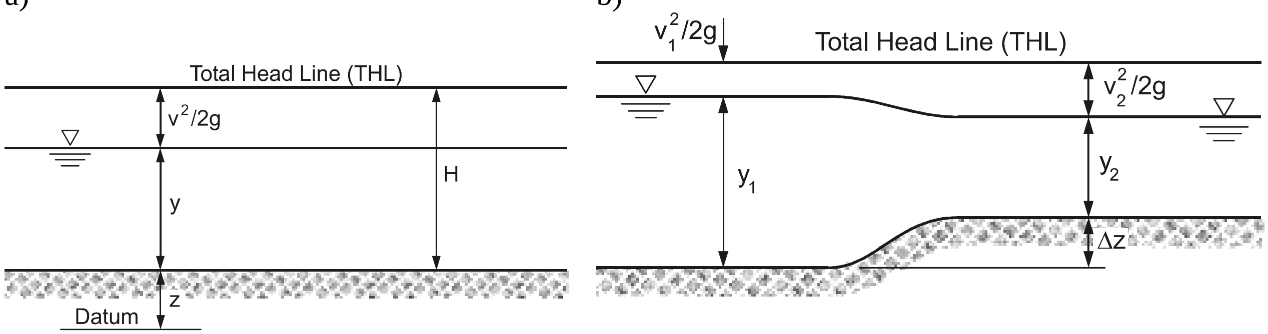


Figure 1. Definition sketch for energy equation

Using continuity equation $Q = V \cdot A$ and assuming that the velocity is constant over the depth over the whole section Equation (1) can be arranged as

$$\frac{Q^2}{2gA_1^2} + y_1 = \frac{Q^2}{2gA_2^1} + y_2 + \Delta z \qquad (2)$$

Introducing the quantity $H = \dfrac{Q^2}{2gA^2} + y$, Equation (2) can be rewritten as

$$H_1 - H_2 = \Delta z \qquad (2.a)$$

The quantity H is called specific energy (specific head) and it can be thought of as the flow total mechanical energy per unit weight with respect to the channel invert in the same cross section. Equation (2) is sufficient for the analysis of any type of open-channel flow transition regardless of the shape of the cross section and the change in the boundary geometry (e.g., bed elevation, channel width) as long as the flow is sensibly uniform before and after the transition. Equation (2) is of great importance in checking the design of hydraulic structures that include boundary transitions provided that the vertical curvatures and accelerations are negligible, and the channel slope is small. Such structures are often used to determine discharges in small channels and flumes (e.g., Parshall flume).

As a typical example, consider a rectangular channel where the flow is obstructed by a change in the bed elevation, as shown in Figure 1.b. Using the discharge per unit width of channel $q = Q/b = vy$ Equation (2) can be written as

$$\frac{q^2}{2gy_1^2} + y_1 = \frac{q^2}{2gy_2^2} + y_2 + \Delta z \qquad (3)$$

For known geometry of the bed transition, Equation (3) allows calculation of the discharge if the depths y_1 and y_2 are known or evaluation of the change in water elevation if the rate of flow and the initial depth in the channel are known. Determination of the water elevation implies solving the cubic equation

$$H = \frac{q^2}{2gy^2} + y \qquad (4)$$

Equation (4) can be solved by a trial-and-error approach. Out of the three solutions of the cubic equation, only two of them (the alternate depths) are physically real as will be discussed next.

If q is arbitrarily held constant Equation (4) permits evaluation of y for any magnitude of H or vice versa, as illustrated by the specific-energy diagram plotted in Figure 2. The diagram shows that for any given specific energy above a certain minimum value (e.g., H_3 = H_4 in Figure 2), two alternate depths are possible, one corresponding to a subcritical flow (y_3) and the other to a supercritical flow (y_4). Other curves might be drawn for different q values. The curves have asymptotes the horizontal (y = 0) and the (H - y) = 0 lines. The minimum specific energy values (H_{c1} and H_{c2} in Figure 2) occur at the critical flow depths (y_{c1} and y_{c2} in Figure 2). By setting equal to zero the derivative of the specific energy, the magnitude of the critical depth can be determined as

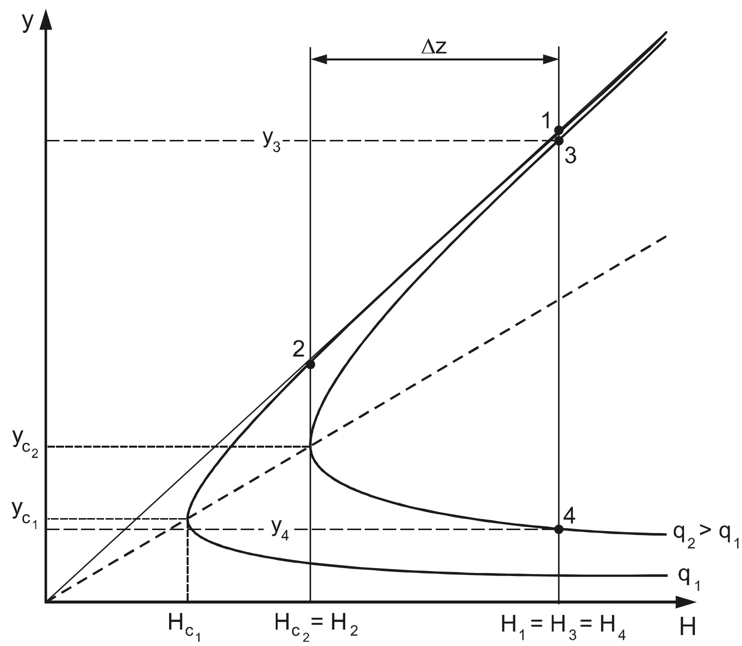


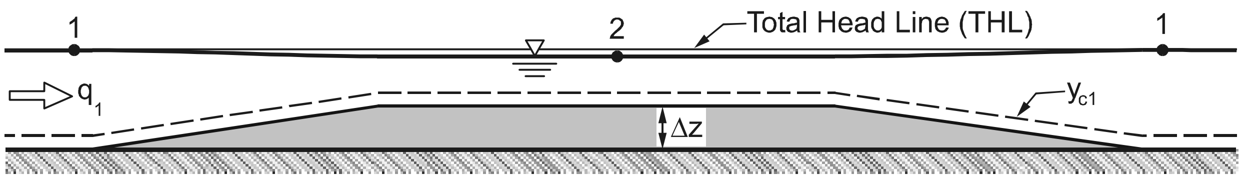
Figure 2. Specific energy diagram

$$y_c = \sqrt[3]{\frac{q^2}{g}} \qquad (5)$$

The behavior of free-surface flows through a rectangular channel can be easily described using the specific energy diagram. It can be observed using the diagram in Figure 2 that if the initial flow depth is larger than the critical, a small decrease in H will result in a decrease in depth; but, if the initial flow depth is smaller than the critical, a small decrease in H will result in an increased depth (Rouse, 1946). Thus, two totally different surface profiles (depending upon whether $y_1 > y_c$ or $y_1 < y_c$) can be produced by a rise in channel boundary, even though the rate of flow and the specific energy are the same for both.

Considered next is the flow situation that will be investigated in our experiment. A steady flow is established through a constant width rectangular channel that has a bed transition (bump), as shown in Figure 1.b. The bump must be smooth and gradual such that streamlines remain nearly parallel (no major energy losses). The unit discharge q_1 is established such that the flow upstream the transition is subcritical (point 1 on the specific energy diagram shown in Figure 2).

a)



b)

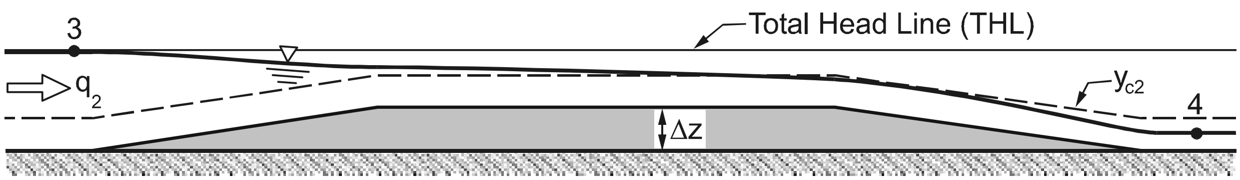


Figure 3. Behavior of the flow over a bump with subcritical flow upstream: a) flow does not attain the critical depth on the bump; b) flow attains critical depth on the bump

According to Equation (2), a bump on the bed creates a decrease in the specific energy. If the height of the bump is such that $\Delta z = H_1 - H_2$ ($H_2 > H_{c2}$), the surprising result is that the flow gets shallower at the bump crest (point 2 in Figure 3.b) and the flow returns to point 1 downstream the bump. If we set another unit discharge $q_2 > q_1$ that maintains a subcritical flow upstream the bump (point 3 on Figure 3.b) and the bump height is such that $\Delta z = H_1 - H_{C2}$, the flow becomes critical at the crest and it changes from subcrititcal to supercritical (3 → C → 4), as shown in Figure 3.b. The objective of this experiment is to observe the changes in the behavior of the free surface when subjected to the above-described situations.

<u>Apparatus</u>

The open channel flume assembly (shown in Figure 4) is located along the southeast wall of IIHR – Hydroscience & Engineering's Model Annex (MA). The flow obstruction used in the experiment is a smooth bump located on the flume floor at about 10 m from the flume entrance. The geometry of the bump is provided in Table 1. For large discharges, the flume flow is generated by a recirculating pump located in a sump at the downstream end of the flume. The discharge in the flume is measured by an orifice-meter located in the return pipe. The calibration equation for the orifice meter is $Q = 1.843 \cdot \Delta h^{.4936}$ (cfs), where Δh is the water level difference (ft of water column) read on the differential manometer connected to the orifice meter. For small discharges a smaller pump (pertaining to the flume filtering system) is used. Anther orifice meter in the discharge line is used to determine discharges. The calibration equation for this orifice meter is $Q = 0.092 \cdot \Delta h^{0.5}$ (cfs), where Δh is the water level difference (ft of water column) read on the differential manometer connected to the orifice meter. Free-surface elevations are measured using a point gage (precision 0.001ft) positioned on an instrumentation carriage moving along the flume. Prior to the experiments, check/set the zero position on the point gage. A measuring tape graduated in cm is glued along the flume rails.

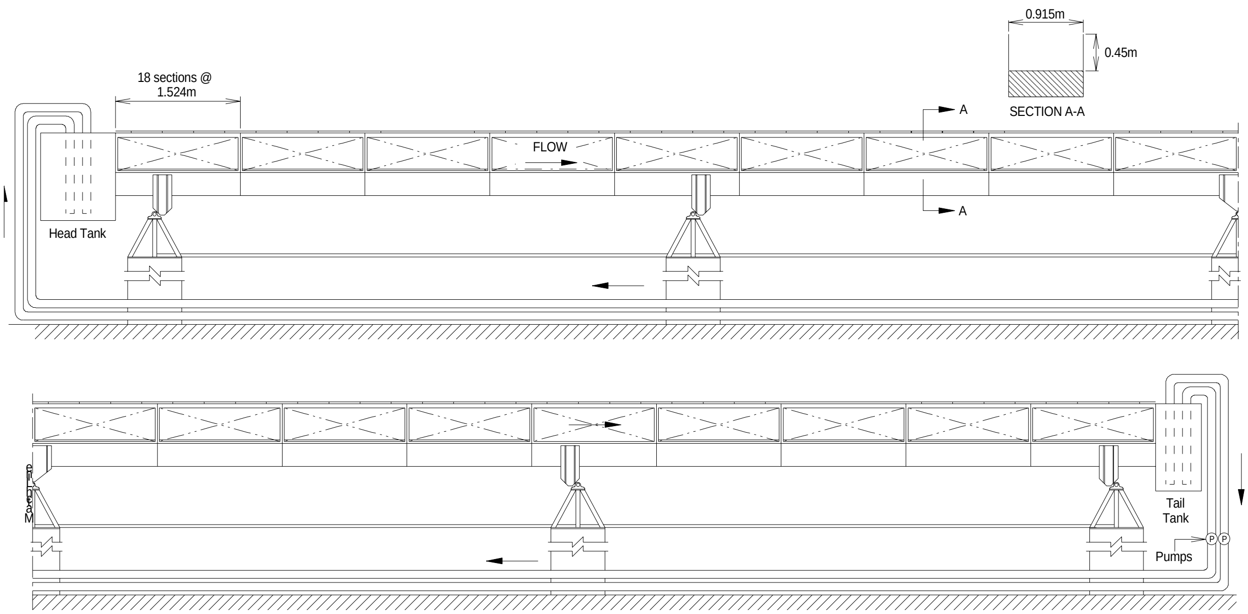


Figure 4. Sketch of the experimental flume

Table 1. Geometry of the flow obstruction

	Equations for bump geometry (m)
	For $0 < x < 0.23$; $y = 0.07x - 0.07$ For $0.23 < x < 0.57$; $y = 0.07$ For $0.57 < x < 0.8$; $y = -0.07x + 1.4$

Procedures

The objective of the experiment is to demonstrate and document two flow situations occurring at the location of the flume transition (bump). Both situations assume subcritical flow upstream the obstruction. Students will measure the governing flow parameters and the free-surface surface elevation in the transition region for the two experimental situations.

1. TAs will set the water depth and pump settings for the flow situation shown in Figure 3.a. Wait for the flow to stabilize.
2. Determine the flow discharge using orifice-meter equation and determine the critical depth using Equation (5).

3. Take measurements of the water level upstream, above, and downstream of the obstruction using intervals of 0.25m. The first measurement should be located at about 0.10m upstream from the bump origin. The location of the measurement along the flume (flume track) will be observed on the measuring tape glued along the flume wall.
4. The flow rate will be changed by the TAs to develop the flow condition shown in Figure 4.b. Wait for the flow to stabilize.
5. Repeat steps 2 and 3. Water level measurements will be taken upstream, above and downstream the bump using intervals of 0.05m. Measurements at intervals of 0.025m should be taken in the region where the critical depth (determined in step 2) is approached.
6. Observe the ripples formed at the tip of the point gage when measurements are made upstream (subcritical flow) and downstream (supercritical flow) the bump.

Measurements

Record your measurements in the table suggested in Appendix A

Data Analysis

For both flow situations conduct analysis using the following steps:

1. Plot the water surface elevation versus the flume track. Locate the bump on the drawing.
2. Determine the actual depth of the flow through the obstruction using the bump geometry provided in Table 1.
3. Plot the specific energy diagram using the actual depth determined above and equation (4).
4. Plot the theoretical specific energy diagram using the measured unit discharge and generated depth values encompassing the interval 0.001 m up to the maximum measured depth.
5. Determine graphically the critical depth on the diagram, compare it with the value obtained from Equation (5) and position it on the graph constructed in step 1. Discuss the comparison of the critical values obtained above and the location of the critical depth on the bump.
6. Identify two alternate depths within the measured depth range and position them on the water surface elevation profile plotted in step 1.
7. Discuss how flow rate affects critical depth and specific energy diagram.

Further considerations

1. What are the main assumptions associated with the use of the specific energy diagram.
2. Comment on the differences noticed on the ripples produced by the point gage upstream and downstream the obstruction.
3. Analyze the water surface behavior for the same unit discharges used in your experiment hypothesizing the flow enters in the obstruction as supercritical flow.

References

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APPENDIX A

Table 2. Measured data

Flow Situation I				**Flow Situation II**			
$\boldsymbol{\Delta h}$ = (ft);	$\boldsymbol{Q}$ = (cfs);	$\boldsymbol{y_{c1}}$ =	(m)	$\boldsymbol{\Delta h}$ = (ft);	$\boldsymbol{Q}$ = (cfs);	$\boldsymbol{y_{c1}}$ =	(m)
Track (m)	Water Surface Elevation (m)	Bump Geometry (m)	Actual depth (m)	Track (m)	Water Surface Elevation (m)	Bump Geometry (m)	Actual depth (m)